2015

(5th Semester)

MATHEMATICS

Paper : MATH–353

(Complex Analysis)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **one** question from each Unit

UNIT—I

1. *(a)* Show that the modulus of sum of two complex numbers is always less than or equal to the sum of their moduli. 5

(b) Find the centre and radius of the circle passing through the points $1, i, 1+i$. 5

(Turn Over)

2. *(a)* If z_1 and z_2 are two complex numbers, then prove that

$$|z_1 - z_2|^2 = |z_1|^2 + |z_2|^2$$

if and only if $z_1\bar{z}_2$ is purely imaginary. 5

(b) If z_1, z_2, z_3 are the vertices of an isoceles triangle, right angled at the vertex z_2, then prove that

$$z_1^2 + z_2^2 + z_3^2 = 2(z_1 + z_3) z_2$$ 5

UNIT—II

3. *(a)* For what value of z the function defined by the equation

$$z = \sin u \cosh v + i \cos u \sinh v, \quad \omega = u + iv$$

ceases to be analytic? 5

(b) Show that the function $f(z) = xy + iy$ is everywhere continuous but not analytic. 5

4. *(a)* If $u = x^3 - 3xy^2$, show that there exists a function $v(x, y)$ such that $\omega = u + iv$ is an analytic function in a finite region. 5

(b) Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin although Cauchy-Riemann equations are satisfied at that point. 5

UNIT—III

5. (a) State and prove Cauchy-Hadamard formula for the radius of convergence. 4

(b) Find the radius of convergence of the following power series : 6

(i) $\sum \frac{z^n}{2^n + 1}$

(ii) $1 + \frac{a \cdot b}{1 \cdot c} z + \frac{a \cdot (a+1) b \cdot (b+1)}{1 \cdot 2 \cdot c \cdot (c+1)} z^2 + \cdots$

6. (a) Find the domain of convergence of the power series

$$\sum \left(\frac{2i}{z+1+i} \right)^n$$ 5

(b) Find the region of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(z+2)^{n-1}}{(n+1)^3 4^n}$$ 5

UNIT—IV

7. (a) Using the definition of the integral of $f(z)$ on a given path, evaluate

$$\int_{-2+i}^{5+3i} z^3 dz$$ 5

(b) If $f(z)$ is analytic within and on a closed contour C and a is any point within C, then show that

$$f(a) = \frac{1}{2\pi i} \int_C \frac{f(z)\, dz}{(z-a)}$$ 5

8. *(a)* Write down the Cauchy's integral formula for the derivative of an analytic function; hence show that for a function $f(z)$ which is analytic in a region D and if $f(z)$ has, at any point $z = a$ of D, derivatives of all orders, all of which are again analytic functions in D, their values are given by

$$f^n(a) = \frac{\lfloor n}{2\pi i}\int_C \frac{f(z)\,dz}{(z-a)^{n+1}}$$

where C is any closed contour in D surrounding the point $z = a$. 6

(b) Evaluate by Cauchy integral formula

$$\int_C \frac{z\,dz}{(9-z^2)(z+i)}$$ 4

UNIT—V

9. *(a)* Expand $\dfrac{z+3}{z(z^2-z-2)}$ for the region $|z| > 2$. 4

(b) Examine the nature of the following functions : 6

(i) $\dfrac{1}{1-e^z}$ at $z = 2\pi i$

(ii) $\dfrac{1}{\sin z - \cos z}$ at $z = \dfrac{\pi}{4}$

10. State and prove Liouville's theorem. Use this result to prove the fundamental theorem of algebra. 1+4+5=10

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2015

(5th Semester)

MATHEMATICS

Paper : MATH–353

(Complex Analysis)

(PART : A—OBJECTIVE)

(*Marks* : 25)

The figures in the margin indicate full marks for the questions

SECTION—A

(*Marks* : 10)

Each question carries 1 mark

Put a Tick ☑ mark against the correct answer in the box provided :

1. If $\bar{z}$ is the conjugate of z, then

(a) $|z| > |\bar{z}|$ ☐

(b) $|z| < |\bar{z}|$ ☐

(c) $|z| = |\bar{z}|$ ☐

(d) $|z| = -|\bar{z}|$ ☐

2. The real part of $\frac{5}{-3+4i}$ is

(a) $\frac{3}{5}$ ☐

(b) $-\frac{3}{5}$ ☐

(c) $-\frac{4}{5}$ ☐

(d) $\frac{4}{5}$ ☐

3. The analytic function whose imaginary part in $e^x \cos y$ is

(a) e^z ☐

(b) ie^z ☐

(c) ie^{-z} ☐

(d) e^{-z} ☐

4. The function $\sin x \cosh y + i \cos x \sinh y$ is

(a) neither continuous nor analytic ☐

(b) continuous but not analytic ☐

(c) continuous as well as analytic everywhere ☐

(d) not analytic everywhere ☐

5. If $\lim_{n \to \infty} |u_r|^{1/n} = l$, then the series Σu_n is absolutely convergent for

(a) $l > 1$ ☐

(b) $l < 1$ ☐

(c) $l = 1$ ☐

(d) $l \geq 1$ ☐

6. The power series $\sum \lfloor n \, z^n$ will converge

(a) if $z = 0$ ☐

(b) if $|z| = 1$ ☐

(c) if $|z| > 1$ ☐

(d) for all real values of z ☐

7. A Jordan curve consisting of continuous chain of a finite number of regular arcs is called a

(a) continuous arc ☐

(b) contour ☐

(c) rectifiable arc ☐

(d) multiple arc ☐

8. The value of the integral $\int_C \frac{dz}{z-a}$, while C is the circle $|z-a|=\rho$ is

(a) 2π ☐

(b) -2π ☐

(c) $2\pi i$ ☐

(d) $-2\pi i$ ☐

9. The function $\frac{z+1}{z(z-2)}$ has/have singularity/singularities at

(a) $z=0$ only ☐

(b) $z=2$ only ☐

(c) $z=0$ and $z=2$ only ☐

(d) $z=-1$ only ☐

10. The nature of the function $\frac{\sin(z-a)}{(z-a)}$ at $z=a$ is

(a) removable singularity ☐

(b) non-isolated singularity ☐

(c) isolated singularity ☐

(d) pole ☐

SECTION—B

(*Marks* : 15)

Each question carries 3 marks

· Answer **all** questions

Answer the following :

1. Prove that for any complex number z, $|z|^2 = z\bar{z}$, where $\bar{z}$ is the conjugate of z.

2. With suitable example, show that continuity is not a sufficient condition for the existence of a finite derivative.

3. Examine the convergence of the series Σz^n.

4. Show that $\int_C \frac{dz}{z} = 2\pi i$, where C is a complete circle

5. Define non-isolated singularity with a suitable example.

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