V/PHY (vi) (PR)

2016

(5th Semester)

PHYSICS

SIXTH PAPER

(Quantum Mechanics—II)

(Pre-Revised)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

1. Explain what you mean by 'matter waves'. Describe with neat diagrams, the experiments of Davisson and Germer on the diffraction of electrons. How does it establish the wave nature of matter? 10

Or

(a) Show that de Broglie wavelength for a material particle of rest mass m_0 and charge q, accelerated from rest through a potential difference of V volts relativistically is given by

$$\frac{h}{\sqrt{2m_0qV\ \ 1\ \ \frac{qV}{2m_0c^2}}}$$ 5

(b) What do you mean by quantum numbers? Write down the possible quantum numbers for n 2. 2

(c) Explain why material particle can only be represented by a group wave, not by a single wave. 3

2. *(a)* What are linear vector space and Hilbert space? 3

(b) Consider three elements from the vector space of real 2 2 matrices

$$|1\ \ \begin{matrix}0 & 1\\ 0 & 0\end{matrix},\ |2\ \ \begin{matrix}1 & 1\\ 0 & 1\end{matrix}\ \text{and}\ |3\ \ \begin{matrix}2 & 1\\ 0 & 2\end{matrix}$$

Show whether they are linearly dependent or not. 3

(c) Use Gram-Schmidt procedure to orthonormalize the basis

$$|1 \quad \begin{matrix}3\\0\\0\end{matrix}, \quad |2 \quad \begin{matrix}0\\1\\2\end{matrix}, \quad |3 \quad \begin{matrix}0\\2\\5\end{matrix}$$ 4

Or

(a) Show that two eigenfunctions of a Hermitian operator belonging to two distinct eigenvalues are orthogonal. 5

(b) Show that if two Hermitian operators commute, their product is also Hermitian. 5

3. Derive time-dependent form of Schrödinger equation. Give a physical interpretation of wave function. Explain the normalization of wave function. 5+2+3=10

Or

State Ehrenfest's theorem. Use this theorem to show that classical mechanics agrees with quantum mechanics so far as expectation values are concerned. 2+8=10

4. What do you mean by quantum mechanical tunnelling effect? Show that the transmittance of a particle incident at rectangular potential barrier is given by

$$T \quad \frac{16E(V_0 \quad E)}{V_0^2} \exp \quad \frac{2\sqrt{2m(V_0 \quad E)}}{h} a$$

where V_0 is the potential barrier, a is barrier thickness. 10

Or

What is zero-point energy of harmonic oscillator? Derive the expression for the eigenfunction in terms of Hermite polynomials of a linear harmonic oscillator. 2+8=10

5. *(a)* Show that commutation relation between X and Y components of angular momentum L_x and L_y is given by

$$[L_x, L_y] \quad i\hbar L_z$$ 5

(b) What do you mean by orbital gyromagnetic ratio for an electron? Obtain an expression for it. 1+4=5

Or

State Uhlenbeck and Goudsmit's hypothesis of electron spin. What are Pauli spin operators? Express Pauli spin functions in the form of 2 2 matrices. Write down the commutation relation satisfied by the three components. 1+3+3+3=10

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Subject Code : V/PHY (vi) (PR)

Booklet No. A

Date Stamp ....

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To be filled in by the Candidate

DEGREE 5th Semester
(Arts / Science / Commerce / ....) Exam., **2016**

Subject ....
Paper ....

INSTRUCTIONS TO CANDIDATES

1. **The Booklet No. of this script should be quoted in the answer script meant for descriptive type questions and vice versa.**
2. **This paper should be ANSWERED FIRST and submitted within 1 (one) Hour of the commencement of the Examination.**
3. **While answering the questions of this booklet, any cutting, erasing, over-writing or furnishing more than one answer is prohibited. Any rough work, if required, should be done only on the main Answer Book. Instructions given in each question should be followed for answering that question only.**

To be filled in by the Candidate

DEGREE 5th Semester
(Arts / Science / Commerce / ....) Exam., **2016**

Roll No. ....
Regn. No. ....

Subject ....
Paper ....

Descriptive Type
Booklet No. B ....

Signature of Scrutiniser(s)

Signature of Examiner(s)

Signature of Invigilator(s)

/129

2016

(5th Semester)

PHYSICS

SIXTH PAPER

(Quantum Mechanics—II)

(Pre-Revised)

(PART : A—OBJECTIVE)

(*Marks* : 25)

The figures in the margin indicate full marks for the questions

SECTION—I

(*Marks* : 10)

Put a Tick (✓) mark against the correct answer in the brackets provided : 1×10=10

1. The ground state energy of hydrogen atom is

(a) – 16·3 eV ( )

(b) – 13·6 eV ( )

(c) – 1·36 eV ( )

(d) – 64·1 eV ( )

2. According to Schrödinger, a particle is equivalent to

(a) a single wave ( )

(b) a wave packet ( )

(c) a light wave ( )

(d) Both *(a)* and *(c)* ( )

3. Two kets $|a\rangle$ and $|b\rangle$ are orthonormal, if

(a) $\langle b|b\rangle = 0$, $\langle a|b\rangle = 0$, $\langle a|a\rangle = 1$ ( )

(b) $\langle a|b\rangle = 1$, $\langle a|a\rangle = 1$, $\langle b|b\rangle = 1$ ( )

(c) $\langle a|b\rangle = 0$, $\langle b|b\rangle = 1$, $\langle a|a\rangle = 1$ ( )

(d) $\langle a|b\rangle = 1$, $\langle b|b\rangle = 1$, $\langle a|a\rangle = 0$ ( )

4. Eigenvalues of Hermitian operators

(a) are real only ( )

(b) are imaginary only ( )

(c) can be real or imaginary ( )

(d) are always complex ( )

5. Let ψ be a wave function, the quantity $\psi^* \psi$ represents

(a) probability density ( )

(b) charge density ( )

(c) energy density ( )

(d) wave density ( )

6. In the equation $F\psi = f\psi$, the eigenfunction of the equation is

(a) F ( )

(b) $f\psi$ ( )

(c) f ( )

(d) ψ ( )

7. For a particle confined in a box, the eigenfunction ψ_n is given by

(a) $\psi_n = A\sin\left(\frac{\pi x}{4an}\right)$ ( )

(b) $\psi_n = A\cos\left(\frac{n\pi x}{3a}\right)$ ( )

(c) $\psi_n = A\sin\left(\frac{n\pi x}{2a}\right)$ ( )

(d) $\psi_n = \sin\left(\frac{n\pi x}{3a}\right)$ ( )

8. For a free particle in step potential, let R and T be reflectance and transmittance, then

(a) $R+T=1$ ( )

(b) $R=T$ ( )

(c) $R-T=1$ ( )

(d) $R \cdot T=1$ ( )

9. For the angular momentum operators L_x and L_y, the operator L_+ can be defined as

(a) $L_+ = L_x + iL_y$ ( )

(b) $L_+ = L_x - iL_y$ ( )

(c) $L_+ = L_y + iL_x$ ( )

(d) $L_+ = L_y - iL_x$ ( )

10. For electron, the number of possible spin states for Z component is

(a) 1 ( )

(b) 2 ( )

(c) 3 ( )

(d) 4 ( )

SECTION—II

(*Marks* : 15)

Give short answers to the following questions : 3×5=15

1. A charge particle accelerated by 200 V has a de Broglie wavelength 0·20 Å. Find the mass of the particle.

2. What is an expectation value of position and momentum operator in quantum mechanics?

3. What are the basic postulates of quantum mechanics?

4. The potential energy of a simple harmonic oscillator of mass m and oscillating with an angular frequency ω is $V(x) = \frac{1}{2} m\omega^2 x^2$. Write down the Hamiltonian operator and Schrödinger equation for the oscillator.

5. Show that electron spin magnetic moment is equal to Bohr magneton.

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