2017

(5th Semester)

PHYSICS

SIXTH PAPER

(Quantum Mechanics—II)

(Revised)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

1. *(a)* What do you mean by 'wave packet'? Derive the expression for group velocity and show that it is equal to particle velocity. 1+4+1=6

(b) Find the relation between group velocity and phase velocity. Hence show that for a non-dispersive medium they are equal. 3+1=4

Or

(a) What do you mean by 'position probability density' denoted by $P(\overset{1}{r},\ t)$? Derive the equation of continuity for probability current density $\vec{J}$. 1+5=6

(b) Discuss two-slit experiment and explain complementary principle. 4

2. *(a)* A particle of energy E is incident on a step potential of height V_0. Show that for $E\quad V_0$, there is a certain probability of being reflected as well as being transmitted and $R\quad T\quad 1$, where R is reflection coefficient and T is transmission coefficient. 6

(b) Discuss the application of quantum tunnelling in   -decay. 4

Or

(a) Show that, $[\hat{p}, \hat{x}]\quad i\hbar$ where $\hat{x}$ and $\hat{p}$ are position and momentum operators respectively. What is the physical significance of this relation? 3+1=4

(b) Show that the eigenvalues of Hermitian operator are real. 4

(c) If $\hat{A}$ and $\hat{B}$ are Hermitian operators, show that $[\hat{A}, \hat{B}]$ is skew-Hermitian operator. 2

3. *(a)* A free particle of mass m moves in a three-dimensional rectangular potential box of sides a, b and c parallel to x-, y- and z-axes respectively. Derive the expression for its normalized wave function and show that the energy eigenvalue for its ground state is given by

$$E_{1,1,1} \quad \frac{{}^2\hbar^2}{2m} \; \frac{1}{a^2} \quad \frac{1}{b^2} \quad \frac{1}{c^2} \qquad 7$$

(b) If the potential box is cubical each of side a, then show that the first excited state is three-fold degenerate. Find the energy eigenvalue for this state. 2+1=3

Or

(a) The ground-state eigenfunction of a simple harmonic oscillator is given by

$$_0(x) \quad Ae^{\;\; a^2x^2/2}$$

where A is a constant and $a^2 \quad m \quad /\hbar$. Find the value of A and write the normalized wave function. Given

$$_0 \; e^{\;\; a^2x^2} dx \quad \frac{\sqrt{}}{2a} \qquad 2$$

(b) Assuming potential energy of the oscillator as $V(x) \quad \frac{1}{2} m \quad {}^2 x^2$, calculate the average value of kinetic energy and potential energy. Given

$$_0 \; x^2 e^{\;\; a^2x^2} dx \quad \frac{\sqrt{}}{4a^3} \qquad 3$$

(c) Write the time-independent Schrödinger equation in the operator form for the simple harmonic oscillator. Hence show that the energy eigenvalue in the ground state is $E_0 \quad \frac{1}{2}\hbar \quad .$ 5

4. *(a)* What do you mean by 'a set of n-vectors, $\vec{a}_1, \vec{a}_2, \ldots, \vec{a}_r$ is linearly independent'? 2

(b) Check whether the following sets of vectors form the basis set in R^3 vector space : 4

$$(2,\ 0,\ \quad 2),\ \ (0,\ 2,\ 0),\ \ (2,\ 0,\ 2)$$

(c) Use the Gram-Schmidt process to transform the basis vectors $u_1 \quad (1,\ 1,\ 1)$, $u_2 \quad (\quad 1,\ 1,\ 0)$ and $u_3 \quad (1,\ 2,\ 1)$ into an orthogonal basis $(v_1,\ v_2,\ v_3)$. 4

Or

(a) What are the conditions to be satisfied by a set of n vectors to form the basis set in an n-dimensional vector space? 2

(b) Consider the following four elements from the vector space of real 2 2 matrices :

$$|1\rangle \quad \begin{matrix} 1 & 0 \\ 0 & 0 \end{matrix} \quad |2\rangle \quad \begin{matrix} 0 & 1 \\ 0 & 0 \end{matrix} \quad |3\rangle \quad \begin{matrix} 0 & 0 \\ 1 & 0 \end{matrix} \quad |4\rangle \quad \begin{matrix} 0 & 0 \\ 0 & 1 \end{matrix}$$

Show whether they form a basis set or not. 4

(c) Write down the condition for orthonormality of state vectors $|\ _m\rangle$ and $|\ _n\rangle$. If

$$|\ _m\rangle \quad 2|u_1\rangle \quad 3|u_2\rangle \quad i|u_3\rangle \text{ and}$$

$$|\ _n\rangle \quad 3|u_1\rangle \quad i|u_2\rangle \quad 4|u_3\rangle$$

find $\langle\ _m|\ _n\rangle$ and $\langle\ _n|\ _m\rangle$. 2+1+1=4

5. *(a)* Starting from the Cartesian components of linear momentum operators, find the Cartesian components of angular momentum operator. 3

(b) Show that

(i) $[L_y, L_z] \quad i\hbar L_x$

(ii) $[L_x, p_x] \quad 0$

where the symbols have their usual meanings. 3+2=5

(c) Suppose we measure the magnitude of angular momentum of a system and find the value, $L^2 \quad 6\hbar^2$. How many orientations of $\vec{L}$ are there with respect to z-axis? What are the corresponding values of L_z? 2

Or

(a) How are the Cartesian components of spin operators $(\hat{S}_x, \hat{S}_y, \hat{S}_z)$ related to their respective Pauli spin operators $(\hat{\ }_x, \hat{\ }_y, \hat{\ }_z)$? Write their corresponding eigenvalues. 2

(b) Show that

(i) $[\ _x, \ _y] \quad 2i \ _z$

(ii) $\ _z \ _x \quad \ _x \ _z \quad 0$ 2+2=4

(c) Taking the value of Pauli spin matrix as

$$_z \quad \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$$

find the values of matrices $\ _x$ and $\ _y$. 4

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Subject Code : PHY/V/06 (R)

Booklet No. A

Date Stamp ....

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To be filled in by the Candidate

DEGREE 5th Semester
(Arts / Science / Commerce / ....) Exam., **2017**

Subject ....

Paper ....

INSTRUCTIONS TO CANDIDATES

1. **The Booklet No. of this script should be quoted in the answer script meant for descriptive type questions and vice versa.**
2. **This paper should be ANSWERED FIRST and submitted within 1 (one) Hour of the commencement of the Examination.**
3. **While answering the questions of this booklet, any cutting, erasing, over-writing or furnishing more than one answer is prohibited. Any rough work, if required, should be done only on the main Answer Book. Instructions given in each question should be followed for answering that question only.**

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DEGREE 5th Semester
(Arts / Science / Commerce / ....) Exam., **2017**

Roll No. ....

Regn. No. ....

Subject ....

Paper ....

Descriptive Type
Booklet No. B ....

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(5th Semester)

PHYSICS

SIXTH PAPER

(Quantum Mechanics—II)

(Revised)

(PART : A—OBJECTIVE)

(*Marks* : 25)

The figures in the margin indicate full marks for the questions

SECTION—I

(*Marks* : 10)

Put a Tick (✓) mark against the correct answer in the brackets provided : 1×10=10

1. The value of the de Broglie wavelength of an electron having kinetic energy of 9 eV is

(a) 4·09 Å ( )

(b) 1·36 Å ( )

(c) 0·4 Å ( )

(d) 13·6 Å ( )

2. Which of the following conditions cannot be satisfied by a well-behaved wave function ψ ?

(a) ψ must be finite for all values of x, y and z ( )

(b) ψ must be single-valued at each point (x, y, z) ( )

(c) ψ must be continuous for all regions ( )

(d) ψ must be a real function of x, y, z, t ( )

3. The quantum mechanical tunnelling provides an explanation for the following physical phenomena except

(a) the emission of alpha particles from a radioactive nucleus ( )

(b) the motion of electrons inside an atom ( )

(c) the electrical breakdown of insulators ( )

(d) the switching action of a tunnel diode ( )

4. If the operator $\hat{A} \equiv \dfrac{d^2}{dx^2}$ operates on the eigenfunction $\psi(x) = \sin 2x$, the eigenvalue is

(a) 1 ( )

(b) 2 ( )

(c) 4 ( )

(d) -4 ( )

5. The total number of energy levels (or degeneracy) for the *n*th state of hydrogen atom is

(a) n ( )

(b) $n+1$ ( )

(c) n^2 ( )

(d) n^2+1 ( )

6. The momentum eigenvalue for a particle trapped in cubical box of side a in the ground state$(1,\ 1,\ 1)$ is

(a) $\dfrac{3\pi\hbar}{a}$ ( )

(b) $\dfrac{\sqrt{3}\pi\hbar}{a}$ ( )

(c) $\dfrac{6\pi\hbar}{a}$ ( )

(d) $\dfrac{\sqrt{6}\pi\hbar}{a}$ ( )

7. If $|\psi\rangle$ and $|\phi\rangle$ are vectors in linear vector spaces and a, b are arbitrary complex numbers, then $\langle a\psi | b\phi\rangle$ is equal to

(a) $ab\langle\psi|\phi\rangle$ ( )

(b) $a^{*}b\langle\psi|\phi\rangle$ ( )

(c) $a^{*}b^{*}\langle\psi|\phi\rangle$ ( )

(d) $ab^{*}\langle\psi|\phi\rangle$ ( )

8. If $|\psi_m\rangle$ and $|\psi_n\rangle$ be two eigenvectors having eigenvalues λ_m and λ_n corresponding to the operator $\hat{\alpha}$, then

(a) $\langle\psi_m|\psi_n\rangle = 0$ ( )

(b) $\langle\psi_m|\psi_n\rangle = 1$ ( )

(c) $\langle\psi_m|\psi_n\rangle = -1$ ( )

(d) $\langle\psi_m|\psi_n\rangle \geq 0$ ( )

9. The Bohr magneton is defined as the magnetic dipole moment associated with an atom due to

(a) orbital motion of an electron in the first stationary orbit ( )

(b) orbital motion of an electron in the first excited state ( )

(c) orbital motion of an electron in presence of magnetic field ( )

(d) electron spin ( )

10. The eigenvectors of $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ are

(a) $\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ ( )

(b) $\begin{pmatrix} 1 \\ i \end{pmatrix}, \begin{pmatrix} 1 \\ -i \end{pmatrix}$ ( )

(c) $\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ ( )

(d) $\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ ( )

SECTION—II

(*Marks* : 15)

Give short answers to the following questions : 3×5=15

1. Find the normalization constant A for the wave function $y(x) = A\sin\frac{npx}{a}$ and hence find the expectation values of x for the particle moving between 0 and a.

2. Show that the momentum operator $\hat{p}_x = -i\hbar \frac{d}{dx}$ is a Hermitian operator.

3. What is the significance of zero-point energy (E_0) in linear harmonic oscillator? Express energy eigen-values (E_n) in terms of E_0.

4. Show that the three vectors $a = (1, 2, 3)$, $b = (3, -1, 1)$ and $c = (1, 1, -2)$ in R^3-space are linearly independent.

5. Explain the concept of electron spin as introduced by Uhlenbeck and Goudsmit.

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