GOVERNMENT ZIRTIRI RESIDENTIAL SCIENCE COLLEGE

Subject: Mathematics

Paper Name: Modern Algebra

Paper Number: 9

Semester: VI

A. <u>Multiple choice questions</u>

1. A subgroup of a group is normal if it is of index
   a) 0
   b) 1
   c) 2
   d) 3
2. If order of a group is p^n, where p is a prime number, then the centre Z of the group is such that
   a) $Z = \{e\}$
   b) $Z \neq \{e\}$
   c) $Z = \phi$
   d) $Z = \{0\}$
3. Let f be a homomorphism of a group G into a group G', then f is an isomorphism of G into G' if
   a) $kerf = \{e\}$
   b) $kerf = G$
   c) $kerf \neq \{e\}$
   d) $kerf \neq G$
4. Let H be a normal subgroup of a group G and K be a normal subgroup of G containing H, then
   a) $G/K \cong \left(\frac{G}{H}\right)/\left(\frac{H}{K}\right)$
   b) $G/K \cong \left(\frac{K}{H}\right)/\left(\frac{G}{H}\right)$
   c) $G/K \cong \left(\frac{G}{H}\right)/\left(\frac{K}{H}\right)$
   d) $G/K \cong \left(\frac{K}{G}\right)/\left(\frac{K}{H}\right)$
5. The set of all self-conjugate elements of a group is called the
   a) Normalizer of the group
   b) Center of the group
   c) Automorphism of the group
   d) Inner automorphism of the group
6. The characteristic of an integral domain is
   a) Either 0 or 1

b) Either 0 or prime
c) Either 1 or prime
d) Necessarily composite

7. A skew field
a) Is commutative
b) Does not possess the unity element
c) Has no divisor of zero
d) Has no invertible elements

8. Which of the following is not an integral domain?
a) The ring of integers
b) The ring $(\{0,1,2,3,4,5\}, +_6, \times_6)$
c) The ring of all 2x2 matrices with elements as integers, the two compositions being addition and multiplication of matrices.
d) The ring $(\{0,1,2,3,4\}, +_5, \times_5)$

9. An ideal S of a commutative ring R with unity is maximal if and only if
a) S is normal in R
b) S is a subring of the ring of residue class R/S
c) the ring of residue class R/S is an integral domain
d) the ring of residue class R/S is a field

10. The ring of rational numbers is
a) An ideal but not a subring of the ring of real numbers
b) A left ideal but not a subring of the ring of real numbers
c) A subring and an ideal of the ring of real numbers
d) A subring but not an ideal of the ring of real numbers

11. Let R and R' be commutative rings with unity. If f be a homomorphism of R into R', the zero elements of R and R' being 0 and $0'$ respectively, then
a) $f(0) = 1$
b) $f(-a) = f(a)$
c) $f(-a) = -f(a)$
d) $f(-a) = f(a^{-1})$

12. The only units of the ring of Gaussian integers are
a) 1 & -1
b) 0 & 1
c) $i \& -i$
d) $1, -1, i, -i$

13. In the quadratic intger ring $Z[i\sqrt{5}]$, 3 is
a) Irreducible but not prime
b) Prime but not irreducible
c) Both prime and irreducible

   d) Neither prime nor irreducible
14. In the ring of integers, 3 & 6 has
   a) Only one gcd
   b) Two gcd
   c) Three gcd
   d) No gcd
15. A non-zero element a in the Euclidean Ring R is a unit in R if
   a) $d(a) > d(1)$
   b) $d(a) < d(1)$
   c) $d(a) = d(1)$
   d) $d(a) = 1$
16. Which of the following is not a subspace of R^3?
   a) $\{(x, y, z): y + z = 0\}$
   b) $\{(x, y, z): \text{y is an integer}\}$
   c) $\{(x, y, z): x - 3y + z = 0\}$
   d) $\{(x, y, z): \text{x is a real number}\}$
17. The linear span of the empty set i.e. $L(\phi)$ is
   a) The whole space itself
   b) The null space, $\{0\}$
   c) ϕ
   d) $\{1\}$
18. Which of the following set of vectors is linearly independent in $V_3(R)$?
   a) $\{(-1,2,1), (3,0,-1), (-5,4,3)\}$
   b) $\{(1,2,0), (0,3,1), (-1,0,1)\}$
   c) $\{(1,2,1), (3,1,5), (3,-4,7)\}$
   d) $\{(4,8,4), (1,2,1)\}$
19. Two subspaces U and W of a vector space V are said to be disjoint if
   a) $U \cup W = \{0\}$
   b) $U \cup W = \emptyset$
   c) $U \cap W = \{0\}$
   d) $U \cap W = \emptyset$
20. Let U be a subspace of a finite dimensional vector space V, then $\dim(V/U) =$
   a) $\dim V + \dim U$
   b) $\dim V - \dim U$
   c) $\dim V \times \dim U$
   d) $\dim V / \dim U$
21. Let U be an n −dimensional vector space over the field F and V be an m −dimensional vector space over the same field F, then the dimension of the vector space $L(U, V)$ of all linear transformations from U into V is

a) $m+n$
b) $m-n$
c) mn
d) m/n

22. Let $T: R^4 \rightarrow R^3$ be a linear transformation defined by $T(x,y,z,t) = (x-y+z+t, x+2z-t, x+y+3z-3t)$. Then the $rank\ (T)$ and $nullity\ (T)$ are respectively
a) 3 & 1
b) 2 & 2
c) 1 & 3
d) 4 & 0

23. Let the function $f: R^3 \rightarrow R^2$ be defined by $f(x,y,z) = (x, y+z)$. Then the kernel of f is given by
a) All (x,y,z) such that $x=0, y=0$
b) All (x,y) such that $x=0, y=0$
c) All (x,y,z) such that $x=0, y=z$
d) All (x,y,z) such that $x=0, y=-z$

24. Let $T: R^3 \rightarrow R^3$ be a linear map defined by $T(x,y,z) = (x+2y-z, y+z, x+y-2z)$. Then the null space of T is
a) $L\{(-3,1,-1)\}$
b) $L\{(-3,-1,-1)\}$
c) $L\{(3,1,-1)\}$
d) $L\{(3,-1,-1)\}$

25. A matrix A represents a one-one transformation if and only if
a) $Rank(A) = 0$
b) $Rank(A) \neq 0$
c) $Nullity(A) = 0$
d) $Nullity(A) \neq 0$

B. <u>Fill in the blanks</u>

1. If order of a group G is __________, where p is a prime number, then G is abelian.
2. Every homomorphic image of a group G is ________________________ to some quotient group of G.
3. Let Z denote the center of a group G. If ________ is cyclic, then G is abelian.
4. Every ____________ integral domain is a field.
5. An ideal of the ring of integers is maximal if and only if it is generated by a ____________________
6. A ring having no proper ideal is called a ____________ ring.
7. Every non-zero element in a Euclidean ring R is either a _____________ or can be written as a product of a finite number of prime elements of R.

8. Let R be an integral domain with unity element 1. Then any two elements of R are relatively prime if their gcd is _________
9. In the ring of integers, the proper divisors of 6 are ________________
10. Every __________________ of a linearly dependent set of vectors is linearly dependent
11. If U and W are subspaces of a finite dimensional vector space V, then $\dim(U + W) =$____
12. Two finite dimensional vector space over the same field are isomorphic if and only if they have the same ___________________________
13. Let V and W be vector spaces over the same field F and let T be a linear transformation from V into W. If V is finite dimensional, then $rank\ (T) +$ ________ $= dim\ V$
14. Let $T: R^3 \to R^3$ be a linear transformation whose matrix with respect to the basis $\{(0,1,1),(1,0,1),(1,1,0)\}$ in R^3 is $\begin{bmatrix} 0 & 3 & 0 \\ 2 & 3 & -2 \\ 2 & -1 & 2 \end{bmatrix}$. Then the matrix of T with respect to the basis $\{(2,1,1),(1,2,1),(1,1,2)\}$ in R^3 is _____________________________
15. If V is finite dimensional, then the nullity of T is the dimension of the _____________________ of T.

Answer key

A. MCQ

1. (c) 2
2. (b) $Z \neq \{e\}$
3. (a) $kerf = \{e\}$
4. (c) $G/K \cong \left(\frac{G}{H}\right)/\left(\frac{K}{H}\right)$
5. (b) Center of the group
6. (b) Either 0 or prime
7. (c) Has no divisor of zero
8. (b) The ring $(\{0,1,2,3,4,5\}, +_6, \times_6)$
9. (d) the ring of residue class R/S is a field
10. (d) subring but not an ideal of the ring of real numbers
11. (c) $f(-a) = -f(a)$
12. (d) $1, -1, i, -i$
13. (a) Irreducible but not prime
14. (b) Two g.c.d

15. (c) $d(a) = d(1)$
16. (b) $\{(x, y, z)\text{: y is an integer}\}$
17. (b) The null space, $\{0\}$
18. (b) $\{(1,2,0), (0,3,1), (-1,0,1)\}$
19. (c) $U \cap W = \{0\}$
20. (b) $\dim V - \dim U$
21. (c) mn
22. (b) 2 & 2
23. (d) All (x, y, z) such that $x = 0, y = -z$
24. (a) $L\{(-3,1,-1)\}$
25. (c) $Nullity(A) = 0$

B. Fill in the blanks

1. p^2
2. Isomorphic
3. G/Z
4. Finite
5. Prime integer
6. Simple
7. Unit
8. 1
9. 2,-2,3,-3
10. Superset
11. $\dim U + \dim W - \dim(U \cap W)$
12. Dimension
13. $nullity(T)$
14. $\begin{bmatrix} -\frac{1}{2} & 2 & \frac{3}{2} \\ \frac{3}{2} & 2 & -\frac{1}{2} \\ \frac{3}{2} & -2 & \frac{7}{2} \end{bmatrix}$
15. Null space