GOVERNMENT ZIRTIRI RESIDENTIAL SCIENCE COLLEGE

Subject : Mathematics
Paper Name : Modern Algebra
Paper No : IX
Semester : VI

A.Multiple Choice Questions:

1. Which of the following statements is false?
   (a) A subgroup *H* of a group *G* is normal if and only if $x^{-1}Hx = H$
   (b) If *H* is a normal subgroup of *G* and *K* is a normal subgroup of *H*, then *K* is a normal subgroup of *G*
   (c) Arbitrary intersection of two normal subgroups is a normal subgroup
   (d) The center *Z* of a group *G* is normal subgroup of *G*

2. If G is a group, the mapping f_a: G → G is an inner automorphism if
   (a) $f_a(x) = ax^{-1}a^{-1}$
   (b) $f_a(x) = a^{-1}xa$
   (c) $f_a(x) = xax^{-1}$
   (d) $f_a(x) = x^{-1}ax$

3. If *f* is a homomorphism of *G* into *G*', then *K* is the kernel of *f* if
   (a) $K = \{ x \in G : f(x) = e' \}$
   (b) $K = \{ x \in G : f(x) = e \}$
   (c) $K = \{ x \in G : f(x) = 0 \}$
   (d) $K = \{ x \in G : f(e) = x \}$

4. If *a* and *b* be two elements of a group *G*, then *b* is conjugate to *a* if
   (a) $b = x^{-1}ax;\ x \in G$
   (b) $b = a^{-1}xa;\ x \in G$
   (c) $b = axa^{-1};\ x \in G$
   (d) $b = xax^{-1};\ x \in G$

5. A subgroup *H* of a group *G* is normal subgroup of G if
   (a) *H* is of index 1 in *G*
   (b) *H* is of index 2 in *G*
   (c) *H* is of index 3 in *G*
   (d) *H* is of index infinity in *G*

6. In the ring of integers *I*, the maximal ideal is
   (a) 6
   (b) 10
   (c) 5
   (d) 8

7. The proper ideals of Z_{12} are < 2 > , < 3 > , < 4 > and < 6 > then the maximal ideals are
   (a) < 2 > and < 4 >
   (b) < 2 > and < 6 >
   (c) < 2 > and < 3 >
   (d) < 4 > and < 6 >

8. The set of all 2×2 matrices of the form $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$, where a , b € I, the set of integers is
   (a) A left ideal in the ring R of all 2×2 matrices with elements as integers
   (b) A right ideal in the ring R of all 2×2 matrices with elements as integers
   (c) An ideal in the ring R of all 2×2 matrices with elements as integers
   (d) A subring and not an ideal in the ring R of all 2×2 matrices with elements as integers

9. The necessary and sufficient conditions for a non-empty subset S of a ring R to be a subring are
   (a) a – b € S and a/b € S for all a, b € S
   (b) a – b € S and ab € S for all a, b € S
   (c) a + b € S and a/b € S for all a, b € S
   (d) a + b € S and ab € S for all a, b € S

10. Which of the following is a ring with zero divisors?
    (a) The ring of integers
    (b) The ring of rational numbers
    (c) $(\{0,1,2,3,4\}, +_5, \times_5)$
    (d) $(\{0,1,2,3,4,5\}, +_6, \times_6)$

11. Let α be a non-zero element in the Euclidean ring *R*, then α is a unit if
    (a) d (α) ≢ d (1)
    (b) d (α) = d (1)
    (c) d (α) < d (1)
    (d) d (α) > d (1)

12. The units in the integral domain *z[i]* are
    (a) 1, -1
    (b) 1, -1, 0, i
    (c) i, -i
    (d) 1, -1, i, -i

13. The units in Z_8 = {0,1,2,3,4,5,6,7} modulo 8 are
    (a) 0, 2, 4, 6
    (b) 1, 3, 5, 6

(c) 1, 3, 5, 7
(d) 4, 5, 6, 7

14. A non-zero integer has
    (a) no associates
    (b) exactly one associate
    (c) exactly two associates
    (d) infinite number of associates

15. In the ring of integers, the greatest common divisor(s) of 3 and 6 is/are
    (a) 3 and -3
    (b) 3
    (c) -3
    (d) 1

16. For the vector space $V_3(F)$ which set is a basis?
    (a) (1,0,0), (1,1,0), (1,1,1)
    (b) (1,0,1), (1,0,0), (0,0,1)
    (c) (1,0,0), (1,1,1)
    (d) (1,0), (0,1)

17. Which of the following statements is false?
    (a) A+B is a subspace of V
    (b) A is a subspace of A+B
    (c) B is a subspace of A+B
    (d) Every element of A+B can be uniquely written in the form a+b, where a € A, b € B and $A \cap B \neq \{0\}$

18. Which of the following sets of vectors is linearly independent in $V_3(R)$?
    (a) {(1,2,0), (0,3,1), (-1,0,1)}
    (b) {(2,1,2), (8,4,8)}
    (c) {(-1,2,1), (3,0, -1), (-5,4,3)}
    (d) {(1,2,1), (3,1,5), (3, -4,7)}

19. The necessary and sufficient condition of a vector space V(F) to be a direct sum of its two subspaces U and W is
    (a) $V = U + W$ and $U \cap W = 0$
    (b) $V = UW$ and $U \cap W = \{0\}$
    (c) $V = U + W$ and $U \cap W \neq \{0\}$
    (d) $V = U + W$ and $U \cap W = \{0\}$

20. Which of the following sets of vectors is linearly dependent?
    (a) {(2, 1, 4), (1, -1, 2), (3, 1, -2)}

(b) {(-1, 2, 1), (3, 0, 1), (-1, 0, 1)}
(c) {(1, 2, 0), (0, 3, 1), (-1, 0, 1)}
(d) {(2, -3,1), (3, -1, 5), (1, -4, 3)}

21. The eigen values of a real symmetric matrix are
    (a) Purely imaginary
    (b) Purely imaginary or zero
    (c) All zero
    (d) All real

22. The eigen values of a real skew-symmetric matrix are
    (a) Purely imaginary
    (b) All zero
    (c) Purely imaginary or zero
    (d) All real

23. An $n \times n$ matrix A over the field F is diagonalizable if and only if
    (a) A has n linearly dependent eigenvectors
    (b) A has n linearly independent eigenvectors
    (c) A has n^2 linearly dependent eigenvectors
    (d) A has n^2 linearly independent eigenvectors

24. If T is a linear transformation from vector space $V_1(F)$ into the vector space $V_2(F)$ and V_1 is finite dimensional of dimension n, then
    (a) rank (T) + nullity (T) = n
    (b) rank (T) + nullity (T) = 1
    (c) rank (T) + nullity (T) = n^2
    (d) rank (T) + nullity (T) = n^n

25. Two eigen vectors of a square matrix A over a field F corresponding to two distinct eigen values are
    (a) Linearly independent
    (b) Linearly dependent
    (c) Inverses of each other
    (d) Equal

B. Fill in the blanks

1. The necessary and sufficient condition for a homomorphism f of a group G with identity e into a group G' with kernel K to be an isomorphism of G into G' is that ___.
2. If the order of a group G with center Z is p^n, where p is a prime number, then ___.
3. A subgroup H of a group G is normal if it is of index ___.
4. A skew field has ___ divisors.
5. The characteristic of the ring $(I_6, +_6, \times_6)$ where $I_6 = \{0,1,2,3,4,5\}$ is ___.

6. The characteristic of the ring of rational numbers is ___.
7. The associates of a non-zero element α + ib of the ring of Gaussian integers D = {α+ib, α, b € I} are ___.
8. The only units in the ring of Gaussian integers are ___.
9. In the quadratic ring of integers $Z[i\sqrt{5}]$ = {α + i√5b; a, b € Z}, the number 3 is ___.

10. Every ___ subset of a finite generated vector space V(F) forms a part of a basis of V.
11. If a finite dimensional vector space V(F) is a direct sum of its two subspaces U and W, then ___.
12. If V(F) is a vector space with zero element 0 and if U and W are disjoint subspaces of V(F), then ___.
13. If A = $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, the eigen values of A are ___.
14. If A and B are similar matrices, then ____.
15. Let T: $R^3 \rightarrow R^3$ be a linear transformation whose nullity is 2. Then the rank of T is ___.

Key answers

Multiple Choice Questions:

1. (a)
2. (b)
3. (a)
4. (a)
5. (b)
6. (c)
7. (c)
8. (d)
9. (b)
10. (d)
11. (b)
12. (d)
13. (c)
14. (c)
15. (b)
16. (a)
17. (d)
18. (a)
19. (d)
20. (b)
21. (d)
22. (c)
23. (b)
24. (a)
25. (a)

Fill in the blanks

1. K = {e}
2. Z $\neq$ {e}
3. 2
4. No zero
5. 6
6. 0
7. α + ib, - α – ib, -b +iα, b – iα
8. 1, -1, i and -i
9. Irreducible but not prime
10. Linearly dependent
11. dim V = dim U/dim W
12. U $\cap$ V = {0}
13. i, -i
14. $|A - \lambda I| = |B - \lambda I|$
15. 1