GOVERNMENT ZIRTIRI RESIDENTIAL SCIENCE COLLEGE

Subject	**:**	**Mathematics**
Paper Name	**:**	**Advanced Calculus**
Paper No	**:**	**X**
Semester	**:**	**VI Semester**

A. Multiple choice questions:

1. If P_1 and P_2 are two partitions of the interval $[a, b]$ and $P_1 \subset P_2$, then
   a) $U(P_1, f) \leq U(P_2, f)$
   b) $U(P_2, f) \leq U(P_1, f)$
   c) $L(P_2, f) \leq L(P_1, f)$
   d) $U(P_2, f) \leq L(P_1, f)$

2. If $f \in R[a, b]$, then
   a) $f^2 \in R[a, b]$
   b) $f^2 \notin R[a, b]$
   c) $f^2 \in R(a, b)$
   d) $f^2 \in R[a, b[$

3. If P and S are any two partitions of $[a, b]$, then
   a) $L(P, f) \leq U(S, f)$
   b) $U(S, f) \leq U(P, f)$
   c) $U(S, f) \leq L(P, f)$
   d) $U(P, f) \geq U(S, f)$

4. If a bounded function *f* is integrable on [a, b], then
   a) $\lim_{\mu(P) \to \infty} S(P, f) = \int_a^b f\,dx$
   b) $\lim_{\mu(P) \to 0} S(P, f) = \int_a^b f\,dx$
   c) $\int_{\underline{a}}^{b} f\,dx = \int_a^b f\,dx$
   d) $L(P, f) = U(P, f) = S(P, f)$
   where $L(P, f), U(P, f)$ and $S(P\ f)$ are the lower Darboux, upper Darboux and Riemann Sum of *f* corresponding to a partition P of [a, b] with norm $\mu(P) < \delta$

5. Let P* be a refinement of a partition P, then for a bounded function *f*
   a) $L(P^*, f) \leq L(P, f)$
   b) $L(P^*, f) \geq L(P, f)$
   c) $U(P^*, f) \leq L(P, f)$
   d) None of the above

6. If *f* and *g* be two positive functions such that $f(x) \leq g(x) \forall \in [a, b]$, then
   a) $\int_a^b g\,dx$ converges if $\int_a^b f\,dx$ converges
   b) $\int_a^b f\,dx$ converges if $\int_a^b g\,dx$ converges
   c) $\int_a^b f\,dx$ diverges if $\int_a^b g\,dx$ diverges
   d) Both (b) and (c) are true.

7. The improper integral $\int_a^\infty \frac{dx}{x^n}$, $a > 0$ converges if and only if

a) $n \leq 1$
b) $n < 1$
c) $n > 1$
d) $n \geq 1$

8. If f and g be two positive functions on [a, b] such that $\lim_{x \to a^+} \frac{f(x)}{g(x)} = l$ a non-zero finite number, then

a) $\int_a^b g\,dx$ converges if $\int_a^b f\,dx$ converges
b) $\int_a^b f\,dx$ converges if $\int_a^b g\,dx$ converges
c) $\int_a^b f\,dx$ diverges if $\int_a^b g\,dx$ diverges
d) $\int_a^b f\,dx$ and $\int_a^b g\,dx$ behave alike

9. The improper integral $\int_a^b \frac{dx}{(x-a)^n}$ converges if and only if

a) $n \leq 1$
b) $n < 1$
c) $n > 1$
d) $n \geq 1$

10. Which of the following definite integrals is an improper integral?

a) $\int_0^{\pi/2} \sin x\,dx$
b) $\int_{-1}^1 \frac{dx}{1+x^2}$
c) $\int_0^4 \frac{dx}{(x-2)(x-3)}$
d) $\int_0^1 \frac{dx}{x(1+x)}$

11. If

$$\int_0^\pi \frac{dx}{a + b\cos x} = \frac{\pi}{\sqrt{a^2 - b^2}}$$

where a is positive and $|b|<a$ then the value of

$$\int_0^\pi \frac{dx}{(a + b\cos x)^2}$$

(a) $\pi/(a-b)$
(b) $\pi/(a^2-b^2$
(c) $\pi/\sqrt{a^2 - b^2}$
(d) $\pi a/\sqrt{a^2 - b^2}$

12. If $\int_0^\infty e^{-\alpha x}\frac{\sin\beta x}{x}dx = \tan^{-1}\frac{\beta}{\alpha}$, then the value of $\int_0^\infty \frac{\sin\beta x}{x}dx = -\frac{\pi}{2}$ if

(a) β<0
(b) β>0
(c) β=0
(d) none of the above

13. The value of the integral $\int_0^\infty \frac{\sin yx}{x(1+x^2)}dx$ is

(a) π/2
(b) $\pi(1-e^{-y})/2$
(c) $\pi e^{-y}/2$
(d) e^{-y}

14. The value of $\int_0^\infty e^{-xy}\cos mx dx$ is

(a) y/m^2
(b) $y/(y^2+m)$
(c) $y/(m^2 + y^2)$
(d) None of the above

15. $\int_0^\infty \frac{e^{-ax}-e^{-bx}}{x}\cos mx\, dx =$

(a) $\log a^2/2$
(b) $\log(m^2+b^2)/2$
(c) $\frac{1}{2}\log\left(\frac{m^2+b^2}{m^2-a^2}\right)$
(d) $\frac{1}{2}\log\left(\frac{m^2+b^2}{m^2+a^2}\right)$

16. The value of $\int_c 4x^3 ds$ where c is the line segment from (-2, -1) to (1, 2), is

(a) $\sqrt{2}$
(b) $3\sqrt{2}$
(c) $-15\sqrt{2}$
(d) $-7\sqrt{3}$

17. Let c be a line joining (0, 1) to (1, 2) then the value of $\int_c (x^2 - y)dx + (y^2 + x)\, dy$ is

(a) 3/5
(b) 5/3
(c) -3/5
(d) -5/3

18. The value of $\int_c \frac{dx}{x+y}$, where c is the curve $x=at^2$, $y=2at$; $0 \le t \le 2$ is
   (a) 2
   (b) log3
   (c) 1
   (d) log4

19. The value of $\iint_A xydxdy$ over the positive quadrant of the circle $x^2 + y^2 = a^2$ is
   (a) $a^4/4$
   (b) a^4
   (c) $a^4/3$
   (d) a^3

20. The value of $\int_c \frac{ds}{x-y}$ along the line $2y=x-4$ between the points (0, -2) and (4, 0) is
   (a) $\sqrt{5}$log2
   (b) log 2
   (c) 5log3
   (d) 2log3

21. The sequence $f_n(x) = nxe^{-nx^2}$ is point-wise convergent on [0, ∞[, but
   (a) not uniformly on [0, ∞[
   (b) uniformly on [0, ∞[
   (c) not uniformly on [0, k[
   (d) none of the above

22. Which of the following statement is correct for the sequence $f_n(x) = \frac{n}{x+n}$?
   (a) convergent in]0, ∞[
   (b) not convergent in [0,1]
   (c) convergent in]-∞,∞[
   (d) not convergent in [0,∞[

23. Let $\{f_n\}$ be such that $\lim_{n\to\infty} f_n(x) = f(x) \forall\, x \in [a,b]$ and let
   $M_n = Sup\{\ |f_n(x) - f(x)| : x \epsilon [a,b]\ \}$. Then $f_n \to f$ uniformly on [a,b] if and only if
   (a) $n \to \infty$ as $M_n \to \infty$
   (b) $M_n \to \infty$ as $n \to \infty$
   (c) $M_n \to 0$ as $n \to \infty$
   (d) none of the above

24. The integration over $0 \le x \le 1$ of the sequence $f_n(x) = 1/(1+nx)$ is
   (a) -1
   (b) 0
   (c) 1
   (d) 2

25. The sequence $f_n(x)=1/(x+n)$ in any interval [0,b] where $b>0$ is
    (a) point-wise convergent only
    (b) uniform convergent
    (c) does not exist
    (d) none of the above

B. Fill in the blanks:

1. The values of $U(P,f)$ and $L(P,f)$ for the function $f(x) = x, 0 \leq x \leq 1$ on taking the partition $P = \left\{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1\right\}$ of [0,1] are _______ and ____

2. For the integral $\int_0^1 x\,dx$, the upper Riemann integral corresponding to the division of [0, 1] into 6 equal interval is _________

3. The definite integral $\int_0^2 \frac{1}{(x-1)(x-2)}\,dx$ is __________

4. The value of the integral $\int_0^\infty e^{-x^2}\,dx$ is _____

5. The improper integral $\int_a^\infty \frac{dx}{x^n}$, $a > 0$ converges if and only if ______

6. The improper integral $\int_0^\infty x^{n-1} e^{-x}\,dx$ is convergent if and only if ________

7. The uniform convergent improper integral of a continuous function is ___________.

8. The value of $f(y) = \int_0^\infty \frac{\cos yx}{1+x^2}\,dx$ is ______________.

9. Let $\emptyset(y) = \int_0^\infty f(x,y)\,dx$ is uniformly convergent, then $\emptyset$ can be integrated under $c \leq y \leq d$ and $x \geq a$ if f be _________.

10. Let A is the region in the xy-plane bounded by the x-axis, the line $y=x$ and $x=\pi$, then $\iint_A \frac{\sin x}{x}\,dA =$________.

11. The value of $\iint_A xy(x+y)\,dx\,dy$ over the area between $y=x^2$ and $y=x$ is______.

12. Let c is a line segment from (0,2) to (1,4). Then $\int_c \sin(\pi y)\,dy + yx^2\,dx =$_______.

13. Every point-wise limit is ________.

14. The sequence $f_n(x) = \frac{nx}{1+n^2x^2}$ converges to f, where $f(x)=0$ $\forall x \epsilon R$, then $f'(x)=$ ____.

15. The value of $\int_0^1 f(x)dx$, where $f_n(x) = n^2x(1-n)^n$, $0 \leq x \leq 1$ is ______.

Answer key:

Multiple choice questions:

A.	**1. (b)**	**2. (a)**	**3. (b)**	**4. (b)**	**5. (b)**
	6. (b)	**7. (c)**	**8. (d)**	**9. (b)**	**10. (d)**
	11. (d)	**12. (a)**	**13.(b)**	**14.(c)**	**15.(d)**
	16.(c)	**17.(b)**	**18.(d)**	**19.(a)**	**20.(a)**
	21. (a)	**22.(d)**	**23.(c)**	**24.(b)**	**25.(b)**

Fill in the blanks:

B.	**1.** $\frac{5}{8}$ **and** $\frac{3}{8}$	**2.** $\frac{7}{12}$	**3. Improper integral of second kind**
	4. $\frac{\sqrt{\pi}}{2}$	**5. n>1**	**6. n>0**
	7. continuous function	**8.** $\frac{\pi}{2}e^{-y}$	**9. Continuous function**
	10. 2	**11. 3/56**	**12. 7/6**
	13. Uniform limit	**14. 0**	**15.0**