V/MAT (vi)

2 0 1 6

(5th Semester)

MATHEMATICS

SIXTH PAPER (MATH–352)

(Real Analysis)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. State and prove Lindelof covering theorem. 2+8=10

2. *(a)* Let $\{G_n\}$ be a sequence of non-empty closed sets such that—

(i) $G_{n \quad 1} \quad G_n \quad n$

(ii) G_1 is bounded

Then prove that the intersection $G_n, \; n \quad N$, is nonempty. 5

(b) Prove that a set is compact if and only if every infinite subset thereof has a limit point in the set. 5

UNIT—II

3. *(a)* Prove that the range of a function continuous on a compact set is compact. 7

(b) If

$$f(x, y) \quad \begin{cases} \dfrac{x^2}{x^2 \quad y^2 \quad x}, & (x, y) \quad (0, 0) \\ 0, & (x, y) \quad (0, 0) \end{cases}$$

then show that f is not continuous at $(0, 0)$. 3

4. *(a)* Prove that a function continuous on a compact domain is uniformly continuous. 6

(b) If

$$f(x, y) \quad \begin{cases} 3xy, & (x, y) \quad (2, 3) \\ 6, & (x, y) \quad (2, 3) \end{cases}$$

then show that f has a removable discontinuity at $(2, 3)$ and redefine the function to make it continuous. 4

UNIT—III

5. *(a)* If $u_1, u_2, u_3, \ldots, u_n$ are functions of $y_1, y_2, y_3, \ldots, y_n$ and $y_1, y_2, y_3, \ldots, y_n$ are functions of $x_1, x_2, x_3, \ldots, x_n$, then prove that

$$\frac{(u_1, u_2, u_3, \ldots, u_n)}{(x_1, x_2, x_3, \ldots, x_n)}$$

$$\frac{(u_1, u_2, u_3, \ldots, u_n)}{(y_1, y_2, y_3, \ldots, y_n)} \quad \frac{(y_1, y_2, y_3, \ldots, y_n)}{(x_1, x_2, x_3, \ldots, x_n)}$$ 6

(b) Prove that a function which is differentiable at a point is also continuous at the point. 4

6. *(a)* Consider the function $f : R^2 \quad R$ defined by

$$f(x, y) \quad \begin{matrix} \dfrac{xy^2}{x^2 \quad y^4}, & \text{if } x \quad 0 \\ 0, & \text{if } x \quad 0 \end{matrix}$$

Show that the directional derivatives of f at (0, 0) in all directions exist but the function is not continuous at (0, 0). 5

(b) Prove that a function which is differentiable at a point admits of partial derivatives at the point. 5

UNIT—IV

7. State and prove Schwarz's theorem. 2+8=10

8. *(a)* Show that for the function

$$f(x, y) \quad \begin{matrix} \dfrac{x^2 y^2}{x^2 \quad y^2}, & (x, y) \quad (0, 0) \\ 0, & (x, y) \quad (0, 0) \end{matrix}$$

$f_{xy}(0, 0) \quad f_{yx}(0, 0)$, even though the conditions of Schwarz's theorem are not satisfied. 6

(b) Examine the function

$$f(x, y) \quad y^2 \quad x^2 y \quad ax^4$$

for extreme values. 4

UNIT—V

9. *(a)* Prove that the space R^n of all ordered n-tuples with the metric d, where

$$d(x, y) \quad \sum_{i \ 1}^{n} (x_i \quad y_i)^2 \ ^{1/2}$$

is a complete metric space. 8

(b) Give an example to show that the intersection of an infinite number of open sets is not open. 2

10. *(a)* Show that every compact subset of a metric space (X, d) is closed. 7

(b) Prove that every closed subset of a compact metric space is compact. 3

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Subject Code : V/MAT (vi)

Booklet No. A

Date Stamp ....

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To be filled in by the Candidate

DEGREE 5th Semester
(Arts / Science / Commerce / ....) Exam., **2016**

Subject ....
Paper ....

INSTRUCTIONS TO CANDIDATES

1. **The Booklet No. of this script should be quoted in the answer script meant for descriptive type questions and vice versa.**
2. **This paper should be ANSWERED FIRST and submitted within 1 (one) Hour of the commencement of the Examination.**
3. **While answering the questions of this booklet, any cutting, erasing, over-writing or furnishing more than one answer is prohibited. Any rough work, if required, should be done only on the main Answer Book. Instructions given in each question should be followed for answering that question only.**

To be filled in by the Candidate

DEGREE 5th Semester
(Arts / Science / Commerce / ....) Exam., **2016**

Roll No. ....
Regn. No. ....

Subject ....
Paper ....

Descriptive Type
Booklet No. B ....

Signature of Scrutiniser(s)

Signature of Examiner(s)

Signature of Invigilator(s)

/142

2016

(5th Semester)

MATHEMATICS

SIXTH PAPER (MATH–352)

(Real Analysis)

(PART : A—OBJECTIVE)

(*Marks* : 25)

Answer **all** questions

SECTION—A

(*Marks* : 10)

Each question carries 1 mark

Put a Tick ☑ mark against the correct answer in the box provided :

1. Every infinite and bounded set has at least one limit point. This is the statement of

(a) Bolzano-Weierstrass theorem ☐

(b) Heine-Borel theorem ☐

(c) Lindelof covering theorem ☐

(d) Cantor intersection theorem ☐

2. If $E \quad R^n$, then a point $a \quad R^n$ is called a limit point of E, if every neighbourhood of the point a contains

(a) an infinite number of points of the set E ☐

(b) a finite number of points of the set E ☐

(c) no point of the set E ☐

(d) None of the above ☐

3. If $\lim_{x \quad a} f(x) \quad b$, where $x \quad (x_1, x_2)$, $a \quad (a_1, a_2)$, $b \quad (b_1, b_2)$ and $f \quad (f_1, f_2)$, then

(a) $\lim_{x \quad a} f_1(x) \quad b_2$ ☐

(b) $\lim_{x \quad a} f_2(x) \quad a_2$ ☐

(c) $\lim_{x \quad a} f_1(x) \quad b_1$ and $\lim_{x \quad a} f_2(x) \quad b_2$ ☐

(d) None of the above ☐

4. A set is said to be compact if it is

(a) bounded ☐

(b) both bounded and closed ☐

(c) open ☐

(d) None of the above ☐

5. If $u \quad \cos x,\ v \quad \sin x \cos y,\ w \quad \sin x \sin y \sin z,$ then

$$\frac{(u, v, w)}{(x, y, z)}$$

is equal to

(a) $\sin^3 x \sin^2 y \sin z$ ☐

(b) $\sin x \sin^2 y \sin^3 z$ ☐

(c) $\quad \sin^3 x \sin^2 y \sin z$ ☐

(d) None of the above ☐

6. If

$$f(x, y) \quad \begin{cases} xy \dfrac{x^2 \quad y^2}{x^2 \quad y^2}, & (x, y) \quad (0, 0) \\ 0, & (x, y) \quad (0, 0) \end{cases}$$

then $f_y(x, 0)$ is equal to

(a) x ☐

(b) y ☐

(c) 0 ☐

(d) None of the above ☐

7. If (a, b) be a point of the domain contained in R^2 of a function f such that f_x and f_y are both differentiable at (a, b), then

(a) $f_{xy}(a, b) \quad f_{yx}(a, b)$ ☐

(b) $f_{xy}(a, b) \quad f_{yx}(a, b)$ ☐

(c) $f_{xy}(a, b) \quad f_{yx}(a, b)$ ☐

(d) None of the above ☐

8. The function $f(x, y) \quad y^2 \quad 4xy \quad 3x^2 \quad x^3$ has

(a) a minimum at (0, 0) ☐

(b) a maximum at (0, 0) ☐

(c) neither a minimum nor a maximum at (0, 0) ☐

(d) None of the above ☐

9. Let (X, d) be a complete metric space and Y be a subspace of X. Then Y is complete if and only if it is

(a) closed in (X, d) ☐

(b) open in (X, d) ☐

(c) both closed and open in (X, d) ☐

(d) None of the above ☐

10. Let A and B be two subsets of a metric space (X, d). Then

(a) $A \quad B \quad \overline{B} \quad \overline{A}$ ☐

(b) $\overline{A \quad B} \quad \overline{A} \quad \overline{B}$ ☐

(c) $\overline{A \quad B} \quad \overline{A} \quad \overline{B}$ ☐

(d) None of the above ☐

SECTION—B

(*Marks* : 15)

Each question carries 3 marks

Answer the following :

1. Show that a set is closed if and only if its complement is open.

2. Define convex set and state the intermediate value theorem.

3. Show that the function f, where

$$f\,(x, y) \quad \begin{cases} \dfrac{x^3 \quad y^3}{x^2 \quad y^2}, & (x, y) \quad (0, 0) \\ 0\,, & (x, y) \quad (0, 0) \end{cases}$$

is continuous at (0, 0).

4. Give an example of a function $f(x, y)$ such that the conditions of Young's theorem are not satisfied but $f_{xy}(0, 0) \quad f_{yx}(0, 0)$. Justify it.

5. In any metric space (X, d), show that the union of an arbitrary family of open sets is open.

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