2016

(5th Semester)

MATHEMATICS

SEVENTH PAPER (MATH–353)

(Complex Analysis)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **one** question from each Unit

UNIT—I

1. *(a)* Prove that

$$\left|\frac{z_1 \quad z_2}{1 \quad \bar{z}_1 z_2}\right| \quad 1$$

if $|z_1| \quad 1$ and $|z_2| \quad 1$. 5

(b) Prove that the area of the triangle whose vertices are the points represented by the complex numbers z_1, z_2, z_3 on the Argand diagram is

$$[(z_2 \quad z_3)|z_1|^2 / 4iz_1]$$ 5

2. *(a)* Find the equations in complex variables of all the circles which are orthogonal to $|z| \quad 1$ and $|z \quad 1| \quad 4$. 5

(b) Find the regions of Argand diagram defined by

$$|z \quad 1| \quad |z \quad 1| \quad 4$$ 5

UNIT—II

3. *(a)* If n is real, then show that

$$r^n(\cos n \quad i\sin n \ )$$

is analytic except when $r \quad 0$ and find its derivatives. 5

(b) If $u \quad e^x(x\cos y \quad y\sin y)$, then find the analytic function $u \quad iv$. 5

4. *(a)* If $f(z) \quad u \quad iv$ is analytic function and $u \quad v \quad e^x(\cos y \quad \sin y)$, then find $f(z)$ in terms of z. 4

(b) If $f(z) \quad u \quad iv$ is analytic function of z in any domain, then prove that

$$\frac{2}{x^2} \quad \frac{2}{y^2} \; |f(z)|^p \quad p^2 |f(z)|^{p \quad 2} |f\ (z)|^2$$ 6

UNIT—III

5. (a) Show that the power series $a_n z^n$ and its derivative $n a_n z^{n \quad 1}$ have same radius of convergence. 5

(b) Find the radius of convergence of the power series

$$f(z) \quad {}_{0} \frac{z^n}{2^n \quad 1}$$

and prove that $(2 \quad z) f(z) \quad 2 \quad 0$ as $z \quad 0$. 5

6. (a) Find the region of convergence of the series

$$ {}_{n \quad 1} \frac{(z \quad 2)^{n \quad 1}}{(n \quad 1)^3 4^n}$$ 5

(b) Find the domain of convergence of the series

$$ {}_{1} \frac{1 \;\; 3 \;\; 5 \; \ldots \; (2n \quad 1)}{\lfloor n} \quad \frac{1 \quad z}{z}^{\,n}$$ 5

UNIT—IV

7. (a) Evaluate

$$ {}_{(0,\,3)}^{(2,\,4)} [(2y \quad x^2)\, dx \quad (3x \quad y)\, dy]$$

using the substitution $x \quad 2t,\ y \quad t^2 \quad 3$. 4

(b) State and prove Cauchy's fundamental theorem. 6

8. (a) Verify Cauchy's theorem for the function $5 \sin 2z$ if C is the square with vertices at $1 \quad i, \quad 1 \quad i$. 5

(b) If $f(z)$ is analytic within and on a closed contour C and a is any point within C, then show that

$$f(a) \quad \frac{1}{2 \quad i} {}_{C} \frac{f(z)\, dz}{(z \quad a)}$$ 5

UNIT—V

9. (a) State and prove Liouville's theorem. 4

(b) State and prove Taylor's theorem. 6

10. *(a)* Obtain the Laurent's series which represents the function

$$f(z) \quad \frac{z^2 \quad 1}{(z \quad 2)(z \quad 3)}$$

in the regions—

(i) $|z| \quad 2$

(ii) $|z| \quad 3$ 5

(b) Find the singularities of the following functions : 5

(i) $\dfrac{\cot \ z}{(z \quad a)^2}$ at $z \quad 0, z$

(ii) $\tan \dfrac{1}{z}$ at $z \quad 0$

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Subject Code : V/MAT (vii)

Booklet No. A

Date Stamp ....

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To be filled in by the Candidate

DEGREE 5th Semester
(Arts / Science / Commerce /
.......................) Exam., **2016**

Subject ....
Paper ....

INSTRUCTIONS TO CANDIDATES

1. **The Booklet No. of this script should be quoted in the answer script meant for descriptive type questions and vice versa.**
2. **This paper should be ANSWERED FIRST and submitted within 1 (one) Hour of the commencement of the Examination.**
3. **While answering the questions of this booklet, any cutting, erasing, over-writing or furnishing more than one answer is prohibited. Any rough work, if required, should be done only on the main Answer Book. Instructions given in each question should be followed for answering that question only.**

To be filled in by the Candidate

DEGREE 5th Semester
(Arts / Science / Commerce /
.......................) Exam., **2016**

Roll No. ....
Regn. No. ....

Subject ....
Paper ....

Descriptive Type
Booklet No. B ....

Signature of Scrutiniser(s)

Signature of Examiner(s)

Signature of Invigilator(s)

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V/MAT (vii)

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(5th Semester)

MATHEMATICS

SEVENTH PAPER (MATH–353)

(Complex Analysis)

(PART : A—OBJECTIVE)

(*Marks* : 25)

SECTION—A

(*Marks* : 10)

Each question carries 1 mark

Put a Tick ☑ mark against the correct answer in the box provided :

1. In an Argand plane, the radius of the circle $|5z \quad 15 \quad 16i| \quad 20$ is

(a) 2 ☐

(b) 20 ☐

(c) 4 ☐

(d) 10 ☐

2. The value of $\arg z \quad \arg \bar{z}$ is

(a) $2n$ ☐

(b) $\quad 2n$ ☐

(c) n ☐

(d) $\quad n$ ☐

3. If $f(z) \quad u \quad iv$ is analytic function in a finite region and $u \quad x^3 \quad 3xy^2$, then v is

(a) $3x^2y^2 \quad y^3 \quad c$ ☐

(b) $3x^2y \quad y^3 \quad c$ ☐

(c) $3x^2y \quad y^2 \quad c$ ☐

(d) None of the above ☐

4. The analytic function whose real part is $e^x \cos y$ is

(a) xe^z ☐

(b) $3e^z$ ☐

(c) e^{2z} ☐

(d) $e^z \quad c\,i$ ☐

5. If the power series $\quad a_n z^n$ is convergent but the series $|a_n z^n|$ is not convergent, then the series $\underset{n \quad 0}{\quad} a_n z^n$ is said to be

(a) conditionally convergent ☐

(b) divergent ☐

(c) oscillatory ☐

(d) None of the above ☐

6. For the series $\quad \frac{n!}{n^2} z^n$, the radius of convergence R is

(a) e ☐

(b) ☐

(c) 1 ☐

(d) 0 ☐

7. If $f(z)$ is analytic in a simply connected domain D enclosed by a rectifiable Jordan curve C and $f(z)$ is continuous on C, then for any point z_0 in D, we have $f(z_0)$

(a) $\frac{1}{2} \quad _C \frac{f(z)\,dz}{z \quad z_0}$ ☐

(b) $\frac{1}{2 \quad i} \quad _C \frac{f(z)\,dz}{z \quad z_0}$ ☐

(c) $2 \quad i \quad \frac{f(z)\,dz}{z \quad z_0}$ ☐

(d) $2 \quad \frac{f(z)\,dz}{z \quad z_0}$ ☐

8. If C is a circle $|z| \;\; 1$, then $\;\; \underset{C}{} \bar{z}\, dz$ is

(a) $\;\; i$ ☐

(b) $2 \;\; i$ ☐

(c) 0 ☐

(d) ☐

9. For the function $f(z) \;\; e^{z}$, $z \;\;\; i$ is

(a) isolated essential singularity ☐

(b) pole ☐

(c) ordinary point ☐

(d) None of the above ☐

10. The number of isolated singular points of

$$f(z) \;\; \frac{z \;\; 3}{z^2(z^2 \;\; 2)}$$

is

(a) 3 ☐

(b) 4 ☐

(c) infinite ☐

(d) 6 ☐

SECTION—B

(*Marks* : 15)

Each question carries 3 marks

Answer **all** questions

Answer the following :

1. Show that for two complex numbers z_1 and z_2

$$|z_1 \quad z_2| \quad |z_1| \quad |z_2|$$

2. Show that the function $f(z)$ xy iy is everywhere continuous but is not analytic.

3. Find the centre and radius of convergence of the power series

$$\frac{(\quad 1)^n}{n}(z \quad 2i)^n$$

4. Evaluate $\quad (\bar{z})^2 dz$ around the circle $|z \quad 1| \quad 1$.

5. Find the zeros and poles of

$$\frac{z \quad 1}{z^{2} \quad 1}^{\;2}$$

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