2017

(5th Semester)

MATHEMATICS

SEVENTH PAPER (MATH–353)

(Complex Analysis)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **one** question from each Unit

UNIT—I

1. *(a)* Show that the modulus of sum of two complex numbers is always less than or equal to the sum of their moduli. 5

(b) Show that arg $\dfrac{z_1 \quad z_2}{z_3 \quad z_4}$ is the angle between the lines joining z_2 to z_1 and z_4 to z_3 on the Argand plane. Also find the conditions if two lines are perpendicular or parallel. 5

2. *(a)* If z_1 and z_2 are two complex numbers, then prove that $|z_1 \quad z_2|^2 \quad |z_1|^2 \quad |z_2|^2$ if and only if $z_1\bar{z}_2$ is purely imaginary. 5

(b) If z_1, z_2, z_3 are the vertices of an isosceles triangle, right angled at the vertex z_2, then prove that

$$z_1^2 \quad 2z_2^2 \quad z_3^2 \quad 2z_2(z_1 \quad z_3)$$ 5

UNIT—II

3. *(a)* Show that the continuity is a necessary but not the sufficient condition for the existence of a finite derivative. 5

(b) Show that the function $f(z) \quad \sqrt{|xy|}$ where, $z \quad x \quad iy$ is not analytic at the origin, although the Cauchy-Riemann equations are satisfied at that point. 5

4. *(a)* If $u \quad x^3 \quad 3xy^2 \quad 3x^2 \quad 3y^2 \quad 1$, then determine harmonic conjugate function and find the corresponding analytic function in terms of z. 5

(b) For what value of z, the function w defined by the following equations ceases to be analytic?

$$z \quad \log \quad i\,;\, w \quad (\cos \quad i\sin \;)$$ 5

UNIT—III

5. *(a)* State and prove Cauchy-Hadamard formula for the radius of convergence. 5

(b) Find the domain of convergence of the series $\sum \left(\frac{iz+1}{2+i}\right)^n$. 5

6. *(a)* Find the radiis of convergence of the following power series : 2×3=6

(i) $\sum \frac{z^n}{2^n+1}$

(ii) $1+\frac{a.b}{1.c}z+\frac{a.(a+1).b.(b+1)}{1.2.c.(c+1)}z^2+\cdots$

(b) If R_1 and R_2 are the radii of convergence of the power series $\sum a_n z^n$ and $\sum b_n z^n$ respectively, then find the radius of the convergence of the power series $\sum a_n b_n z^n$. 4

UNIT—IV

7. *(a)* Find the value of the integral—

$$\int_0^{1+i}(x-y+ix^2)\,dz$$

(i) along the straight line from $z=0$ to $z=1+i$;

(ii) along the real axis from $z=0$ to $z=1$ and then along a line parallel to the imaginary axis from $z=1$ to $z=1+i$. 5

(b) Show that if a function $f(z)$ is analytic in a region D, then its derivative at any point $z=a$ of D is also analytic in D, and is given by

$$f'(a)=\frac{1}{2\pi i}\int_C \frac{f(z)\,dz}{(z-a)^2}$$

where C is any closed contour in D surrounding the point $z=a$. 5

8. *(a)* Evaluate by Cauchy integral formula

$$\int_C \frac{z\,dz}{(9-z^2)(z+i)}$$

where C is the circle $|z|=2$. 5

(b) If $f(z)$ is a continuous function in a domain D and if for every closed contour C in the domain D $\int_C f(z)\,dz=0$, then prove that $f(z)$ is analytic within D. 5

UNIT—V

9. *(a)* Obtain the Laurent's series which represents the function

$$\frac{1}{(1+z^2)(z+2)}$$

in the region $|z|>2$. 5

(b) Define singularity of a complex function. With suitable example, explain the terms isolated and non-isolated singularities. 5

10. *(a)* Find the singularities of the following functions : 4

(i) $\sin \dfrac{1}{1 \quad z}$ at $z \quad 1$

(ii) $\operatorname{cosec} \dfrac{1}{z}$ at $z \quad 0$

(b) State and prove the maximum modulus theorem. 6

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Subject Code : MATH/V/07

Booklet No. A

Date Stamp ....

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To be filled in by the Candidate

DEGREE 5th Semester
(Arts / Science / Commerce / ....) Exam., **2017**

Subject ....

Paper ....

INSTRUCTIONS TO CANDIDATES

1. **The Booklet No. of this script should be quoted in the answer script meant for descriptive type questions and vice versa.**
2. **This paper should be ANSWERED FIRST and submitted within 1 (one) Hour of the commencement of the Examination.**
3. **While answering the questions of this booklet, any cutting, erasing, over-writing or furnishing more than one answer is prohibited. Any rough work, if required, should be done only on the main Answer Book. Instructions given in each question should be followed for answering that question only.**

To be filled in by the Candidate

DEGREE 5th Semester
(Arts / Science / Commerce / ....) Exam., **2017**

Roll No. ....

Regn. No. ....

Subject ....

Paper ....

Descriptive Type
Booklet No. B ....

Signature of Scrutiniser(s)

Signature of Examiner(s)

Signature of Invigilator(s)

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MATH/V/07

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(5th Semester)

MATHEMATICS

SEVENTH PAPER (MATH–353)

(Complex Analysis)

(PART : A—OBJECTIVE)

(*Marks* : 25)

SECTION—A

(*Marks* : 10)

Each question carries 1 mark

Put a Tick ☑ mark against the correct answer in the box provided :

1. The modulus of $\exp(1 \quad i)$ is

(a) e ☐

(b) $1/e$ ☐

(c) 1 ☐

(d) $\quad e$ ☐

2. If $\bar{z}$ is the conjugate of z, then

(a) $|z| \quad |\bar{z}|$ ☐

(b) $|z| \quad |\bar{z}|$ ☐

(c) $|z| \quad |\bar{z}|$ ☐

(d) $|z| \qquad |\bar{z}|$ ☐

3. The derivative of $e^x(\cos y \quad i\sin y)$ is

(a) e^x ☐

(b) e^y ☐

(c) $e^{x \quad iy}$ ☐

(d) $e^{x \quad iy}$ ☐

4. The analytic function whose imaginary part is $e^x \cos y$ is

(a) e^z ☐

(b) ie^z ☐

(c) $ie^{\quad z}$ ☐

(d) $e^{\quad z}$ ☐

5. If $\lim_{n} \quad |u_n|^{1/n} \quad l$, then the series $\quad u_n$ is convergent if

(a) $l \quad 1$ ☐

(b) $l \quad 1$ ☐

(c) $l \quad 1$ ☐

(d) $l \quad 1$ ☐

6. The power series $\quad n!z^n$ will converge

(a) if $z \quad 0$ ☐

(b) if $|z| \quad 1$ ☐

(c) if $|z| \quad 1$ ☐

(d) for all real values of z ☐

7. If C is given by the equation $|z \quad a| \quad R$, then the value of $\quad {}_C \frac{dz}{z \quad a}$ is

(a) $2 \quad i$ ☐

(b) $\quad i$ ☐

(c) $\quad 2 \quad i$ ☐

(d) $\quad i$ ☐

8. A continuous arc without multiple points is called a

(a) Jordan arc ☐

(b) continous arc ☐

(c) contour ☐

(d) rectifiable arc ☐

9. The function $\frac{\sin(z \quad a)}{(z \quad a)}$ at $z \quad a$ has

(a) removable singularity ☐

(b) non-isolated singularity ☐

(c) isolated singularity ☐

(d) a pole ☐

10. Zeros of the function $\frac{z^2 \quad 4}{e^z}$ at $z \qquad$ are

(a) $2i$ ☐

(b) $\quad 2i$ ☐

(c) $\quad 2i$ ☐

(d) z ☐

SECTION—B

(*Marks* : 15)

Each question carries 3 marks

Answer **all** questions

Answer the following :

1. If $a^2 \quad b^2 \quad 1$, then find the value of

$$\frac{1 \quad a \quad ib}{1 \quad a \quad ib}$$

2. If $w \quad f(z)$ is an analytic function, $z \quad x \quad iy$, show that

$$\frac{dw}{dz} \quad (\cos \quad i \sin \quad) \frac{w}{r}$$

where $x \quad r \cos \quad , y \quad r \sin \quad .$

3. Examine the convergence of the series z^n.

4. Evaluate $\int_{2\ \ i}^{5\ \ 3i} z^3 dz$

5. Give the statements of Taylor's and Laurent's theorems.

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