V / MAT (viii) (B)

2015

(5th Semester)

MATHEMATICS

PAPER : MATH–354(B)

(Probability Theory)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* Show that if an event A is independent of the events B, $B \cap C$ and $B \cup C$, then it is also independent of C. 5

(b) Prove that for any two events A and B

$$P(A \cap B) \leq P(A) \leq P(A \cup B) \leq P(A) + P(B) \qquad 5$$

2. State and prove Bayes' theorem. 10

UNIT—II

3. For the binomial distribution $(q+p)^n$, prove that

$$\mu_{r+1} = pq\left(nr\mu_{r-1} + \frac{d\mu_r}{dp}\right)$$

where μ_r is the rth central moment. Hence obtain μ_2, μ_3 and μ_4. Also find out β_1 and β_2. 10

4. *(a)* The mean and the variance of a binomial distribution are 4 and $\frac{4}{3}$ respectively. Find *(i)* the probability of two successes and *(ii)* the probability of more than two successes. 5

(b) The probability distribution function of a random variable X is

$$f(x) = \begin{cases} x, & \text{for } 0 < x \le 1 \\ 2-x, & \text{for } 1 \le x < 2 \\ 0, & \text{for } x \ge 2 \end{cases}$$

Compute the cumulative distribution function of X. 5

UNIT—III

5. *(a)* For the following bivariate probability distribution of X and Y, find the following : 6

(i) $P(X \le 1, Y = 2)$

(ii) $P(X \le 1)$

(iii) $P(Y = 3)$

(iv) $P(Y \le 3)$

(v) $P(X < 3, Y \le 4)$

$Y \rightarrow$ / $X \downarrow$	1	2	3	4	5	6
0	0	0	$\frac{1}{32}$	$\frac{2}{32}$	$\frac{2}{32}$	$\frac{3}{32}$
1	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
2	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$	0	$\frac{2}{64}$

(b) The joint density function of X, Y is given as

$$f(x, y) = 2, \quad 0 < x < y < 1$$
$$= 0, \quad \text{elsewhere}$$

Examine whether X and Y are independent or not. 4

6. *(a)* The joint probability density function of two random variables X and Y is given by

$$f(x, y) = \frac{9(1 + x + y)}{2(1 + x)^4 (1 + y)^4}, \quad \begin{array}{l} 0 < x < \infty \\ 0 < y < \infty \end{array}$$

UNIT—III

5\. (a) For the following bivariate probability distribution of X and Y, find the following : 6

(i) $P(X \leq 1, Y = 2)$

(ii) $P(X \leq 1)$

(iii) $P(Y = 3)$

(iv) $P(Y \leq 3)$

(v) $P(X < 3, Y \leq 4)$

$Y \rightarrow$ / $X \downarrow$	1	2	3	4	5	6
0	0	0	$\frac{1}{32}$	$\frac{2}{32}$	$\frac{2}{32}$	$\frac{3}{32}$
1	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
2	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$	0	$\frac{2}{64}$

(b) The joint density function of X, Y is given as

$$f(x, y) = 2, \quad 0 < x < y < 1$$
$$= 0, \quad \text{elsewhere}$$

Examine whether X and Y are independent or not. 4

6\. (a) The joint probability density function of two random variables X and Y is given by

$$f(x, y) = \frac{9(1 + x + y)}{2(1 + x)^4 (1 + y)^4}, \quad \begin{array}{l} 0 < x < \infty \\ 0 < y < \infty \end{array}$$

(Turn Over)

Obtain *(i)* marginal distribution of X and Y, and *(ii)* conditional distribution of Y for $X = x$. 6

(b) If X and Y are two independent continuous random variables, then find the probability distribution function of their product. 4

UNIT—IV

7. *(a)* A sample of 3 items is selected at random from a box containing 12 items of which 3 are defectives. Find the expected number of defective items. 7

(b) If X is a random variable, then prove that

$$\text{var}(X) = E(X^2) - [E(X)]^2$$ 3

8. Calculate the correlation coefficient for the following heights (in inches) of fathers (X) and their sons (Y) : 10

X	:	65	66	67	67	68	69	70	72
Y	:	67	68	65	68	72	72	69	71

UNIT—V

9. *(a)* Find the moment generating function of Poisson distribution. 3

(b) Find the moment generating function of exponential distribution. 3

(c) Define gamma distribution and then find the first four moments about the origin. 4

10. For a normal distribution, show that its even order central moments are given by the relation (about mean)

$$\mu_{2n} = (2n-1)(2n-3)\ldots 3.1.\sigma^{2n}$$ 10

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2015

(5th Semester)

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PAPER : MATH–354(B)

(Probability Theory)

(PART : A—OBJECTIVE)

(*Marks* : 25)

SECTION—A (Multiple choice)

(*Marks* : 10)

Each question carries 1 mark

Answer **all** questions

Put a Tick ☑ mark against the correct answer in the box provided :

1×10=10

1. The conditional probability of B given A is

(a) $\dfrac{P(A \cap B)}{P(B)}$ ☐

(b) $\dfrac{P(A \cup B)}{P(A)}$ ☐

(c) $\dfrac{P(A \cap B)}{P(A)}$ ☐

(d) $P(A)\,P(B)$ ☐

2. Let A and B be events with $P(A) = \frac{3}{8}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{4}$, then $P(A \cup B)$ is

(a) $\frac{5}{8}$ ☐

(b) $\frac{1}{2}$ ☐

(c) $\frac{3}{8}$ ☐

(d) $\frac{3}{4}$ ☐

3. The parameters of a binomial distribution with mean 8 and variance 4 are

(a) $n = 2,\ p = \frac{1}{4}$ ☐

(b) $n = 16,\ p = \frac{1}{2}$ ☐

(c) $n = 32,\ p = \frac{1}{4}$ ☐

(d) None of the above ☐

4. The characteristic function of binomial distribution is

(a) $(q + pe^{it})^n$ ☐

(b) $(q + pe^{it})$ ☐

(c) $(qp - e^{it})^n$ ☐

(d) None of the above ☐

5. For two random variables X and Y, the relation $E(XY) = E(X)E(Y)$ holds good

(a) for all X and Y ☐

(b) if X and Y are identical ☐

(c) if X and Y are independent ☐

(d) None of the above ☐

6. If var $(X) = 2$, then var $(3X \pm 5)$ is equal to

(a) 13 ☐

(b) 18 ☐

(c) 5 ☐

(d) –1 ☐

7. The mean of a normal distribution is 50, its mode will be

(a) 25 ☐

(b) 40 ☐

(c) 50 ☐

(d) All of the above ☐

8. The mean and variance of exponential distribution with parameter θ are

(a) $\frac{1}{\theta}, \frac{1}{\theta}$ ☐

(b) $\frac{1}{\theta}, \frac{1}{\theta^2}$ ☐

(c) θ, θ^2 ☐

(d) None of the above ☐

9. The relationship between mean and variance of gamma distribution is

(a) mean = variance ☐

(b) mean = 2 variance ☐

(c) mean < variance ☐

(d) None of the above ☐

10. The mean and variance of the geometric distribution are

(a) $\frac{p}{q}, \frac{q}{p^2}$ ☐

(b) $\frac{p}{q}, \frac{p^2}{q}$ ☐

(c) $\frac{q}{p}, \frac{q}{p^2}$ ☐

(d) $\frac{q}{p}, \frac{p^2}{q}$ ☐

SECTION—B (Very short answer)

(*Marks* : 15)

Each question carries 3 marks

Answer **all** questions

1. If A and B are independent events, then $\bar{A}$ and $\bar{B}$ are also independent events. Prove it.

2. Is it possible to have a binomial distribution with mean 2 and variance 6?

3. Show that if X has Poisson distribution with parameter λ, then $E(X) = \lambda$.

4. Find the moment generating function of geometric distribution.

5. Prove that the moment generating function of gamma distribution is

$$M_X(t) = (1-t)^{-n}, \quad |t| < 1$$

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