2 0 1 7

(5th Semester)

MATHEMATICS

Paper : MATH–354(B)

(Probability Theory)

Full Marks : 75

Time : 3 hours

(Part : B—descriptive)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

Unit—I

1. *(a)* If A, B and C are random events in a sample space where A, B and C are pair-wise independent and A is independent of $B \quad C$, then prove that A, B and C are mutually independent. 5

(b) A speaks the truth in 60% and B in 75% of the cases. In what percentage of the cases are they likely to contradict each other in starting the same fact? 5

2. *(a)* Prove that for any two events A and B,

$$P(A \quad B) \quad P(A) \quad P(A \quad B) \quad P(A) \quad P(B)$$ 5

(b) From a vessel containing 3 white and 5 black balls, 4 balls are transferred into an empty vessel. From this vessel a ball is drawn and is found to be white. What is the probability that out of the four balls transferred 3 are white and 1 is black? 5

Unit—II

3. *(a)* Two dice are rolled. Let X denote the random variable which counts the total number of points on the upturned faces. Construct a table giving the non-zero values of probability mass function. Also, find the distribution function of X. 5

(b) Let X be a continuous random variable with probability density function

$$f(x) \quad \begin{matrix} ax, & 0 \quad x \quad 1 \\ a, & 1 \quad x \quad 2 \\ ax \quad 3a, & 2 \quad x \quad 3 \\ 0, & \text{elsewhere} \end{matrix}$$

(i) Determine the constant a.

(ii) Compute $P(X \quad 1 \ 5)$. 5

4. For a binomial variate x with parameters n and p, prove that

$$_{r\quad 1} \quad pq \;\; nr \quad _{r\quad 1} \quad \frac{d \quad _r}{dp}$$

where $_r$ is the rth central moment. Hence obtain $_2$, $_3$ and $_4$. 10

UNIT—III

5. *(a)* Prove that two random variables X and Y with joint probability density function $f(x, y)$ are stochastically independent if and only if $f_{XY}(x, y)$ can be expressed as the product of a non-negative function of x alone and a non-negative function of y alone, that is, if $f_{XY}(x, y) \quad h_X(x) \;\; k_Y(y)$, where $h(\ ) \quad 0$ and $k(\ ) \quad 0$. 6

(b) A random observation on a bivariate population (X, Y) can yield one of the following pairs of values with probabilities noted against them :

For each observation pair	*Probability*
(1, 1); (2, 1); (3, 3); (4, 3)	1/20
(3, 1); (4, 1); (1, 2); (2, 2); (3, 2); (4, 2); (1, 3); (2, 3)	1/10

Examine if the two events $X \quad 4$ and $Y \quad 2$ are independent. 4

6. *(a)* The joint probability distribution of two random variables X and Y given below :

X / Y	0	1	2
0	3/28	9/28	3/28
1	3/14	3/14	0
2	1/28	0	0

Find—

(i) the marginal probability distribution of X and Y;

(ii) the conditional distribution of X, given the value of $Y \quad 0$. 5

(b) If X and Y are two random variables having joint density function

$$f(x, y) \quad \begin{matrix} \frac{1}{8}(6 \quad x \quad y), & 0 \quad x \quad 2, 2 \quad y \quad 4 \\ 0, & \text{otherwise} \end{matrix}$$

find—

(i) $P(X \quad 1 \quad Y \quad 3)$;

(ii) $P(X \quad Y \quad 3)$;

(iii) $P(X \quad 1 \,|\, Y \quad 3)$. 5

UNIT—IV

7. Two unbiased coins are thrown. If X is the sum of the numbers showing up, prove that

$$P(|X \quad 7| \quad 3) \quad \frac{35}{54}$$

by using Chebychev's inequality. 10

8. Prove that correlation coefficient is independent of the change of origin and scale. 10

UNIT—V

9. *(a)* Define geometric distribution for a random variable X. Find its mean and variance. 7

(b) Find the moment generating function of exponential distribution. 3

10. *(a)* If X and Y are independent Poisson variates, calculate the conditional distribution of X, given $X \quad Y$ is binomial. 5

(b) For a normal distribution, prove that mean = median. 5

★★★

Subject Code : MATH/V/08b

Booklet No. A

Date Stamp ....

....

To be filled in by the Candidate

DEGREE 5th Semester
(Arts / Science / Commerce /
.......................) Exam., **2017**

Subject ....
Paper ....

INSTRUCTIONS TO CANDIDATES

1. **The Booklet No. of this script should be quoted in the answer script meant for descriptive type questions and vice versa.**
2. **This paper should be ANSWERED FIRST and submitted within 1 (one) Hour of the commencement of the Examination.**
3. **While answering the questions of this booklet, any cutting, erasing, over-writing or furnishing more than one answer is prohibited. Any rough work, if required, should be done only on the main Answer Book. Instructions given in each question should be followed for answering that question only.**

To be filled in by the Candidate

DEGREE 5th Semester
(Arts / Science / Commerce /
.......................) Exam., **2017**

Roll No. ....
Regn. No. ....

Subject ....
Paper ....

Descriptive Type
Booklet No. B ....

Signature of Scrutiniser(s)

Signature of Examiner(s)

Signature of Invigilator(s)

/226

MATH/V/08b

2 0 1 7

(5th Semester)

MATHEMATICS

Paper : MATH–354(B)

(Probability Theory)

(PART : A—OBJECTIVE)

(*Marks* : 25)

SECTION—A

(*Marks* : 10)

Each question carries 1 mark

Put a Tick ☑ mark against the correct answer in the box provided : 1×10=10

1. If $p_1 \quad P(A)$, $p_2 \quad P(B)$, $p_3 \quad P(A \quad B)$, then $P(\overline{A \quad B})$ in terms of p_1, p_2, p_3 is

(a) $1 \quad p_1 \quad p_2 \quad p_3$ ☐

(b) $1 \quad p_1 \quad p_2 \quad p_3$ ☐

(c) $1 \quad p_1 \quad p_3$ ☐

(d) $1 \quad p_1 \quad p_3$ ☐

2. A coin is tossed three times in succession, the number of sample points in sample space is

(a) 6 ☐

(b) 8 ☐

(c) 3 ☐

(d) 9 ☐

3. Let X be a random variable having discrete uniform distribution over the range $[1, n]$. Then the variance $V(X)$ is given by

(a) $\frac{(n \quad 1)}{2}$ ☐

(b) $\frac{(n \quad 1)(2n \quad 1)}{6}$ ☐

(c) $\frac{(n \quad 1)(n \quad 1)}{12}$ ☐

(d) $\frac{(n \quad 1)(2n \quad 1)}{6}$ ☐

4. For the probability mass function $f(x) \quad cx^2(1 \quad x)$, $0 \quad x \quad 1$, the value of the constant c is

(a) 4 ☐

(b) 8 ☐

(c) 12 ☐

(d) 0 ☐

5. The marginal probability function of a continuous random variable X is defined as

(a) $f_X(x) \qquad \underset{y}{} p_{XY}(x, y)$ ☐

(b) $f_X(x) \qquad \underset{x}{} p_{XY}(x, y)$ ☐

(c) $f_X(x) \qquad f_{XY}(x, y)\, dx$ ☐

(d) $f_X(x) \qquad f_{XY}(x, y)\, dy$ ☐

6. If X and Y are two random variables, the covariance between them is defined as

(a) $\mathrm{Cov}\,(X, Y) \quad E(XY) \quad E(X)E(Y)$ ☐

(b) $\mathrm{Cov}\,(X, Y) \quad E(XY) \quad E(X)E(Y)$ ☐

(c) $\mathrm{Cov}\,(X, Y) \quad E(X) \quad E(Y) \quad E(XY)$ ☐

(d) $\mathrm{Cov}\,(X, Y) \quad E(X)E(Y) \quad E(XY)$ ☐

7. If X is a random variable, then

(a) $\mathrm{Var}\,(X) \quad \{E(X)\}^2 \quad E(X^2)$ ☐

(b) $\mathrm{Var}\,(X) \quad E\,(X^2) \quad \{E(X)\}^2$ ☐

(c) $\mathrm{Var}\,(X) \quad E(X) \quad E(X^2)$ ☐

(d) $\mathrm{Var}\,(X) \quad E(X^2) \quad E(X)$ ☐

8. If X and Y are independent random variables, then

(a) Cov (X, Y) $\quad E(XY) \quad E(X)E(Y)$ ☐

(b) Cov (X, Y) $\quad E(X)E(Y) \quad E(XY)$ ☐

(c) Cov (X, Y) $\quad 1$ ☐

(d) Cov (X, Y) $\quad 0$ ☐

9. Normal curve is

(a) very flat ☐

(b) bell-shaped symmetrical about mean ☐

(c) very peaked ☐

(d) smooth ☐

10. The relationship between mean and variance of Gamma distribution is

(a) mean = variance ☐

(b) mean = 2 variance ☐

(c) mean < variance ☐

(d) mean > variance ☐

SECTION—B

(*Marks* : 15)

Each question carries 3 marks

1. A letter is known to have come either from TATANAGAR or from CALCUTTA. On the envelope just two consecutive letters TA are visible. What is the probability that the letter came from CALCUTTA?

2. A continuous random variable X has a probability density function $f(x) \quad 3x^2, 0 \quad x \quad 1$. Find a and b such that

(a) $P(X \quad a) \quad P(X \quad a)$

(b) $P(X \quad b) \quad 0 \;\; 05$

3. The joint probability density function of a two-dimensional random variable (X, Y) is given by

$$f(x, y) \quad \begin{matrix} 2, & 0 \quad x \quad 1, \quad 0 \quad y \quad x \\ 0, & \text{elsewhere} \end{matrix}$$

Find the marginal density functions of X and Y.

4. Let X be a random variable with the following probability distribution :

x	:	-3	6	9
$P(X \quad x)$	:	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

Find $E(X)$ and $E(X^2)$ and using the laws of expectation, evaluate $E(2X \quad 1)^2$.

5. Prove that the sum of two independent Poisson variates is again a Poisson variate.

★★★