2016

(1st Semester)

MATHEMATICS

FIRST PAPER

(Calculus)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **one** question from each Unit

UNIT—I

1. *(a)* Draw the graph of the function defined by

$$f(x) = \begin{cases} x^2 & ; \text{ when } x < 0 \\ x & ; \text{ when } 0 \le x < 1 \\ \frac{1}{x} & ; \text{ when } x \ge 1 \end{cases}$$

Discuss whether $f(x)$ is continuous at $x = 1$. 5

(b) If $y = \cos(m \sin^{-1} x)$, then show that

$$(1 - x^2)y_{n+2} = (2n+1)xy_{n+1} + (n^2 - m^2)y_n$$

and find $y_n(0)$. 5

2. *(a)* Using ϵ - δ definition, show that

$$\lim_{x \to \frac{\pi}{6}} \sin x = \frac{1}{2}$$ 5

(b) Evaluate : 5

(i) $\displaystyle \lim_{x \to 0} \frac{e^x - e^{-x} - 2x}{x - \sin x}$

(ii) $\displaystyle \lim_{x \to 2} \frac{x^3 - 8}{x - 2}$

UNIT—II

3. *(a)* State and prove Rolle's theorem. 6

(b) Show that for $0 < x < \frac{\pi}{2}$

$$\frac{2}{\pi} < \frac{\sin x}{x} < 1$$ 4

4. *(a)* Expand $\cos x$ in an infinite series in powers of x and hence show that

$$\sin^2 x = x^2 - \frac{1}{3}x^4 + \frac{2}{9}x^6 - \cdots \quad \forall x \in R$$ 6

(b) Find the intervals in which

$$f(x) \quad 2x^3 \quad 15x^2 \quad 36x \quad 1$$

is monotonic increasing or monotonic decreasing. 4

UNIT—III

5. *(a)* Evaluate : 3+3

(i) $\dfrac{dx}{x\sqrt{1 \quad x^3}}$

(ii) $(x \quad 1)\sqrt{x^2 \quad x \quad 1}\, dx$

(b) Evaluate ${}_a^b e^x\, dx$ from first principle. 4

6. *(a)* Evaluate

$${}_0^{/2} \sin^n x\, dx$$

where n is a positive integer. 5

(b) Prove that

$${}_0^{/2} \frac{x\, dx}{\sec x \quad \operatorname{cosec} x} \quad \frac{}{4} \quad 1 \quad \frac{1}{\sqrt{2}} \log(\sqrt{2} \quad 1)$$ 5

UNIT—IV

7. *(a)* If $u \quad x^2 \tan^{\ 1} \dfrac{y}{x} \quad y^2 \tan^{\ 1} \dfrac{x}{y}, \quad xy \quad 0,$ then show that

$$\frac{{}^2 u}{x \quad y} \quad \frac{x^2 \quad y^2}{x^2 \quad y^2}$$ 6

(b) Let $f : R^2 \quad R$ be a function defined by

$$f(x, y) \quad \begin{matrix} \dfrac{x^2 y}{x^4 \quad y^2} & ; & (x, y) \quad (0, 0) \\ 0 & ; & (x, y) \quad (0, 0) \end{matrix}$$

Test the continuity of f at $(0, 0)$. 4

8. *(a)* If $V \quad \log_e \dfrac{x^3 \quad y^3}{x^2 \quad y^2}$, then show that

$$x \frac{V}{x} \quad y \frac{V}{y} \quad 1$$ 5

(b) Find the area included between the curves $y^2 \quad 4ax$ and $x^2 \quad 4ay$. 5

UNIT—V

9. *(a)* Prove that a monotonic increasing sequence which is bounded above is convergent. 5

(b) Show that the sequence $\{S_n\}$, where

$$S_n \quad \frac{1^3}{n^4} \quad \frac{2^3}{n^4} \quad \cdots \quad \frac{n^3}{n^4}$$

converges to $\frac{1}{4}$. 5

10. (a) Show that the series

$$\frac{1}{1^p} \quad \frac{1}{2^p} \quad \frac{1}{3^p} \quad \cdots$$

is divergent for $p \quad 1$. 5

(b) Test the convergence of the series

$$1 \quad \frac{1}{2.3} \quad \frac{1.3}{2.4.5} \quad \frac{1.3.5}{2.4.6.7} \quad \cdots$$ 5

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Subject Code : I/ MAT (i)

Booklet No. A

Date Stamp ....

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To be filled in by the Candidate

DEGREE 1st Semester
(Arts / Science / Commerce /
.......................) Exam., **2016**

Subject ....

Paper ....

INSTRUCTIONS TO CANDIDATES

1. **The Booklet No. of this script should be quoted in the answer script meant for descriptive type questions and vice versa.**
2. **This paper should be ANSWERED FIRST and submitted within 1 (one) Hour of the commencement of the Examination.**
3. **While answering the questions of this booklet, any cutting, erasing, overwriting or furnishing more than one answer is prohibited. Any rough work, if required, should be done only on the main Answer Book. Instructions given in each question should be followed for answering that question only.**

To be filled in by the Candidate

DEGREE 1st Semester
(Arts / Science / Commerce /
.......................) Exam., **2016**

Roll No. ....

Regn. No. ....

Subject ....

Paper ....

Descriptive Type
Booklet No. B ....

Signature of Scrutiniser(s)

Signature of Examiner(s)

Signature of Invigilator(s)

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I/ MAT (i)

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(1st Semester)

MATHEMATICS

FIRST PAPER

(Calculus)

(PART : A—OBJECTIVE)

(*Marks* : 25)

Answer **all** questions

SECTION—I

(*Marks* : 10)

Each question carries 1 mark

Put a Tick ☑ mark against the correct answer in the box provided :

1. If $f(x) \quad 2f(1 \quad x) \quad x^2 \quad 2 \quad x \quad R$, then $f(x)$ is given by

(a) $\dfrac{(x \quad 2)^2}{3}$ ☐

(b) $x^2 \quad 2$ ☐

(c) 1 ☐

(d) None of the above ☐

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2. If $y \quad x^{n \quad 1} \log x$, then xy_n is equal to

(a) $\underline{|n}$ ☐

(b) $\underline{|n \quad 1}$ ☐

(c) $\underline{|n \quad 2}$ ☐

(d) None of the above ☐

3. If $f(x) \quad \sin x \quad \cos x$, then $f\ (x) \quad 0$ has a root in the interval

(a) $0, \frac{}{2}$ ☐

(b) $\frac{}{2},$ ☐

(c) $\frac{}{2}, 0$ ☐

(d) $\frac{}{4}, \frac{3}{4}$ ☐

4. $f(x) \quad e^x$ can be expanded in powers of $(x \quad 2)$ by using

(a) Maclaurin's theorem ☐

(b) Taylor's theorem ☐

(c) Leibnitz's theorem ☐

(d) Euler's theorem ☐

5. $_{0}^{/2} \dfrac{\sin x}{\sin x \quad \cos x}\, dx$ is equal to

(a) $\dfrac{}{4}$ ☐

(b) $\dfrac{}{2}$ ☐

(c) 0 ☐

(d) None of the above ☐

6. $_{0}^{2} [x]\, dx$ is equal to

(a) 2 ☐

(b) 0 ☐

(c) 1 ☐

(d) None of the above ☐

7. If $z \quad xy\, f \; \dfrac{x}{y}$, then $x \dfrac{z}{x} \quad y \dfrac{z}{y}$ is equal to

(a) 0 ☐

(b) z ☐

(c) $\dfrac{1}{z}$ ☐

(d) $2z$ ☐

8. $\lim\limits_{(x,\ y)\quad (1,\ 1)} \dfrac{\sin (xy \quad 1)}{\tan (2xy \quad 2)}$ is equal to

(a) 2 ☐

(b) $\frac{1}{2}$ ☐

(c) 0 ☐

(d) None of the above ☐

9. The sequence $\{u_n\}$, where $u_n \quad 1 \quad (\quad 1)^n$ is

(a) convergent ☐

(b) bounded but not convergent ☐

(c) unbounded ☐

(d) None of the above ☐

10. The series $1 \quad \frac{1}{2^2} \quad \frac{1}{3^2} \quad \frac{1}{4^2} \quad \ldots$ is

(a) convergent ☐

(b) divergent ☐

(c) oscillating ☐

(d) None of the above ☐

SECTION—II

(*Marks* : 15)

Each question carries 3 marks

1. Evaluate $\lim_{x} \; 1 \;\; \frac{1}{x} \;\; ^{x}$.

2. Show that the equation $\cos x \quad x \sin x \quad 0$ has a root in the interval $0, \frac{\ }{2}$.

3. If $y \quad {}_{x}^{x^{2}} \sin u \; e^{u} \; du$, then find $\dfrac{dy}{dx}$.

4. Find the volume of the solid bounded by the surface $z \quad \sqrt{1 \quad x^2 \quad y^2}$ and the plane $z \quad 0$.

5. Prove that

$$\lim_{m} \frac{1^m \quad 2^m \quad 3^m \quad \cdots \quad n^m}{n^{m \quad 1}} \quad \frac{1}{m \quad 1}; \; m \qquad 1$$

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