2017

(2nd Semester)

MATHEMATICS

SECOND PAPER

(Algebra)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* If A is a square matrix of order n and I is unit matrix of order n, then prove that

$$A(\text{adj } A) \quad |A|I \quad (\text{adj } A)A$$ 5

(b) Reduce the matrix

$$\begin{matrix} 0 & 1 & 3 & 1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & 2 & 0 \end{matrix}$$

to the normal form and hence obtain its rank. 5

2. *(a)* Find the inverse of the matrix

$$\begin{matrix} 9 & 7 & 6 \\ 7 & 1 & 8 \\ 3 & 4 & 2 \end{matrix}$$

by elementary operations. 5

(b) Determine whether the following system of linear equations is consistent or not, then solve the equations : 5

$$\begin{matrix} 5x & 3y & 7z & 4 \\ 3x & 26y & 2z & 9 \\ 7x & 2y & 10z & 5 \end{matrix}$$

UNIT—II

3. *(a)* Let S be the set of all real numbers except –1. Define on S by

$$a \quad b \quad a \quad b \quad ab$$

Show that is *(i)* commutative and *(ii)* associative. 2½+2½=5 *(b)* Prove that the inverse of the product of two elements of a group G is the product of the inverses taken in the reverse order. 5

4. *(a)* Prove that every subgroup of a cyclic group is cyclic. 5

(b) Let H be a subgroup of G. Let the relation ~ be defined on G by $a \sim b$ if and only if ab^{1} H. Then prove that—

(i) ~ is an equivalence relation;

(ii) ~ decomposes G into right cosets of H in G. 2½+2½=5

Unit—III

5. *(a)* State and prove Euler's extension of Fermat's theorem and apply it to show that the remainder on dividing 7^9 by 15 is 7. 6

(b) Let $f : G \quad G$ be a group homomorphism. Prove that $\ker f \quad \{e\}$, if and only if f is an isomorphism. 4

6. *(a)* Define isomorphism of a group. If R is the additive group of real numbers and R is the multiplicative group of all positive real numbers, then prove that $f : R \quad R$ defined by $f(x) \quad e^x$ for all $x \quad R$ is an isomorphism of R onto R . 2+4=6

(b) Prove that the order of every element of a finite group is a divisor of the order of the group. 4

Unit—IV

7. *(a)* If a polynomial $f(x)$ is divided by

$$(x \quad a)(x \quad b), \; a \quad b$$

then prove that the remainder is

$$\frac{(x \quad b) f(a) \quad (x \quad a) f(b)}{a \quad b}$$ 5

(b) Prove that $x^4 \quad x^2 \quad 1$ is a factor of $x^{12} \quad 1 \quad 0$. 5

8. *(a)* Find the remainder, when

$$x^5 \quad 3x^4 \quad 4x^2 \quad x \quad 4$$

is divided by $(x \quad 1)(x \quad 2)$. 4

(b) If a polynomial $f(x)$ be divided by a binomial $(x \quad h)$, then show that the remainder is $f(h)$. 3

(c) Find the value of a in order that the expression $4x^4 \quad (a \quad 1)x^3 \quad ax^2 \quad 6x \quad 1$ may be divisible by $(2x \quad 1)$. 3

UNIT—V

9. *(a)* Solve the equation $x^3 \quad 3x \quad 1 \quad 0$ by Cardan's method. 6

(b) If , , are the roots of the equation

$$x^3 \quad px^2 \quad qx \quad r \quad 0$$

then find the value of $\frac{\quad ^2 \quad\quad ^2}{\qquad\qquad}$. 4

10. *(a)* The equation $ax^3 \quad 3bx^3 \quad 3cx \quad d \quad 0$ has two equal roots. Prove that

$$(bc \quad ad)^2 \quad 4(b^2 \quad ac)(c^2 \quad bd)$$ 5

(b) If , , are the roots of the equation

$$x^3 \quad px^2 \quad qx \quad r \quad 0$$

then find the equation whose roots are

$$\frac{1}{\ }, \qquad \frac{1}{\ }, \qquad \frac{1}{\ }$$ 5

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Subject Code : MATH/II/02

Booklet No. A

Date Stamp ....

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To be filled in by the Candidate

DEGREE 2nd Semester
(Arts / Science / Commerce /
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Subject ....
Paper ....

INSTRUCTIONS TO CANDIDATES

1. **The Booklet No. of this script should be quoted in the answer script meant for descriptive type questions and vice versa.**
2. **This paper should be ANSWERED FIRST and submitted within 1 (one) Hour of the commencement of the Examination.**
3. **While answering the questions of this booklet, any cutting, erasing, over-writing or furnishing more than one answer is prohibited. Any rough work, if required, should be done only on the main Answer Book. Instructions given in each question should be followed for answering that question only.**

To be filled in by the Candidate

DEGREE 2nd Semester
(Arts / Science / Commerce /
.......................) Exam., **2017**

Roll No. ....
Regn. No. ....

Subject ....
Paper ....

Descriptive Type
Booklet No. B ....

Signature of Scrutiniser(s)

Signature of Examiner(s)

Signature of Invigilator(s)

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(2nd Semester)

MATHEMATICS

SECOND PAPER

(Algebra)

(PART : A—OBJECTIVE)

(*Marks* : 25)

SECTION—A

(*Marks* : 10)

Each question carries 1 mark

Put a Tick ☑ mark against the correct answer in the box provided :

1. The rank of matrix $A \begin{matrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 2 & 2 \end{matrix}$ is

(a) 2 ☐

(b) 1 ☐

(c) 0 ☐

(d) 3 ☐

2. If A and B are Hermitian, then $AB \quad BA$ is Hermitian and $AB \quad BA$ is

(a) Hermitian ☐

(b) skew-Hermitian ☐

(c) symmetric ☐

(d) skew-symmetric ☐

3. The number of binary composition on a finite set A having n elements is

(a) n^{n^2} ☐

(b) 2^{n^2} ☐

(c) n^n ☐

(d) $n!$ ☐

4. The number of generators of a cyclic group of order 8 is

(a) 16 ☐

(b) 4 ☐

(c) 1 ☐

(d) 32 ☐

5. When 99^{20} is divided by 25, then the remainder is

(a) 20 ☐

(b) 5 ☐

(c) 15 ☐

(d) 1 ☐

6. A homomorphism of a group into itself is called

(a) an isomorphism ☐

(b) kernel of a homomorphism ☐

(c) an endomorphism ☐

(d) an automorphism ☐

7. The expression x^5 $61x$ p is divisible by x 1. The value of p is

(a) 62 ☐

(b) 60 ☐

(c) 60 ☐

(d) 0 ☐

8. If a polynomial $f(x)$ is divided by $(x \quad a)$ and if the remainder $R \quad f(a) \quad 0$, then $(x \quad a)$ is a factor of

(a) $f(a)$ ☐

(b) $f(x)$ ☐

(c) $(x \quad a)$ ☐

(d) a ☐

9. The equation $x^{12} \quad x^4 \quad x^3 \quad x^2 \quad 1 \quad 0$ has

(a) at least six complex roots ☐

(b) two real roots and four complex roots ☐

(c) at least six real roots ☐

(d) three real roots and three complex roots ☐

10. The de Moivre's form of complex number $3 \quad 4i$ is

(a) $5(\cos \quad i\sin \quad)$ ☐

(b) $5(\cos \quad i\sin \quad)$ ☐

(c) $3(\cos \quad i\sin \quad)$ ☐

(d) $4(\cos \quad i\sin \quad)$ ☐

SECTION—B

(*Marks* : 15)

Each question carries 3 marks

State *True* or *False* of the following with a brief justification :

1. The rank of a skew-symmetric matrix is greater than or equal to 2.

True ☐ *False* ☐

Justification :

2. In a group $(G, \quad)$, the elements a and b commute, then $a^{\;1}$ and b also commute.

True ☐ *False* ☐

Justification :

3. If $f : G \quad G$ is a homomorphism and $f(G)$ is the homomorphism image of G in G, then $f(G)$ is a subgroup of G.

True ☐ *False* ☐

Justification :

4. x 1 is a root of the equation x^3 $3x$ 2 0 of multiplicity 2.

True ☐ *False* ☐

Justification :

5. The roots of a cubic equation are 2 $\quad i$, 2 $\quad i$ and 3. The equation is $x^3 \quad 7x^2 \quad 17x \quad 15 \quad 0$.

True ☐ *False* ☐

Justification :

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