2017

(CBCS)

(2nd Semester)

MATHEMATICS

SECOND PAPER

(Algebra)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **one** question from each Unit

UNIT—I

1. *(a)* State Cayley-Hamilton theorem. Find the inverse of

$$A \quad \begin{matrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{matrix}$$

using characteristic polynomial. 1+4=5

(b) Solve the system of the following equations : 5

$$\begin{matrix} x & y & z & 9 \\ 2x & 5y & 7z & 52 \\ 2x & y & z & 0 \end{matrix}$$

2. *(a)* Prove that every non-singular matrix is invertible. 5

(b) Reduce the matrix

$$\begin{matrix} 0 & 1 & 3 & 1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & 2 & 0 \end{matrix}$$

to be normal form and hence obtain its rank. 5

UNIT—II

3. *(a)* Show that the set of residue classes modulo m is an Abelian group of order m with respect to addition of residue classes. 5

(b) Show that the set of four permutations I, $(1\ 2)(3\ 4)$, $(1\ 3)(2\ 4)$ and $(1\ 4)(2\ 3)$ on four symbols 1, 2, 3, 4 is an Abelian group with respect to the permutation multiplication. 5

4. *(a)* If H is a subgroup of a group $(G, \circ)$, then show that $HH^{1} \quad H$. 5

(b) Show that the order of a cyclic group is equal to the order of its generator. 5

UNIT—III

5. *(a)* Find the regular permutation group isomorphic to the multiplicative group $(1, \quad 1, i, \quad i)$. 5

(b) State and prove Fermat's theorem. 5

6. *(a)* Show that the group $(\{1, 2, 3, 4, 5, 6\}, X_7)$ is cyclic. How many generators are there? 4

(b) If $f : G \quad G$ be a group homomorphism, then show that for all $a \quad G$, order of a order of $f(a)$. 6

UNIT—IV

7. *(a)* If a polynomial $f(x)$ is divided by $(x \quad)(x \quad)$, $\quad$, then prove that the remainder is

$$\frac{(x \quad)f(\quad) \quad (x \quad)f(\quad)}{\quad}$$ 5

(b) By Euclidean algorithm, find the greatest common divisor of 26 and 118. Also find the integers x and y to satisfy $d \quad 26x \quad 118y$. 3+2=5

8. *(a)* State and prove division algorithm. 1+5=6

(b) Expand $2x^4 \quad x^3 \quad 2x^2 \quad 5x \quad 1$ in powers of $(x \quad 3)$. 4

UNIT—V

9. *(a)* State the fundamental theorem of algebra. Prove that a polynomial $f(x)$ of degree n, $n \quad 1$, with coefficients in $\mathbb{R}$ or $\mathbb{C}$ has exactly n roots. 1+5=6

(b) If the equation

$$3x^4 \quad 4x^3 \quad 60x^2 \quad 96x \quad k \quad 0$$

has four real roots, then show that k must lie between 32 and 43. 4

10. *(a)* If the equation $ax^3 \quad 3bx^2 \quad 3cx \quad d \quad 0$ has two equal roots, then prove that $(bc \quad ad)^2 \quad 4(b^2 \quad ac)(c^2 \quad bd)$. 5

(b) Solve the equation $x^3 \quad 3x \quad 1 \quad 0$ by Cardan's method. 5

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Subject Code :
MATH/II/EC/02 (CBCS)

Booklet No. A

Date Stamp ....

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To be filled in by the Candidate

CBCS
DEGREE 2nd Semester
(Arts / Science / Commerce / ....) Exam., **2017**

Subject ....
Paper ....

INSTRUCTIONS TO CANDIDATES

1. **The Booklet No. of this script should be quoted in the answer script meant for descriptive type questions and vice versa.**
2. **This paper should be ANSWERED FIRST and submitted within 1 (one) Hour of the commencement of the Examination.**
3. **While answering the questions of this booklet, any cutting, erasing, over-writing or furnishing more than one answer is prohibited. Any rough work, if required, should be done only on the main Answer Book. Instructions given in each question should be followed for answering that question only.**

To be filled in by the Candidate

CBCS
DEGREE 2nd Semester
(Arts / Science / Commerce / ....) Exam., **2017**

Roll No. ....
Regn. No. ....

Subject ....
Paper ....

Descriptive Type
Booklet No. B ....

Signature of Scrutiniser(s)

Signature of Examiner(s)

Signature of Invigilator(s)

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MATH/II/EC/02 (CBCS)

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(CBCS)

(2nd Semester)

MATHEMATICS

SECOND PAPER

(Algebra)

(PART : A—OBJECTIVE)

(*Marks* : 25)

Answer **all** questions

SECTION—A

(*Marks* : 10)

Each question carries 1 mark

Put a Tick ☑ mark against the correct answer in the box provided :

UNIT—I

1. If A is a square matrix, then $A \quad A^t$ is

(a) skew-symmetric ☐

(b) symmetric ☐

(c) Hermitian ☐

(d) None of the above ☐

2. If A is $m \quad n$ R-echelon matrix with r non-zero rows, then the row rank of A is

(a) m ☐

(b) n ☐

(c) r ☐

(d) $m \quad n$ ☐

UNIT—II

3. If Q be the set of all positive real numbers and a binary operation on Q defined by $a \quad b \quad \frac{ab}{3}$, then the identity element in Q is

(a) 1 ☐

(b) 2 ☐

(c) 3 ☐

(d) $\frac{1}{3}$ ☐

4. Which of the following permutations is even permutation?

(a) $(1\ 2\ 3)\ (1\ 2)$ ☐

(b) $(1\ 2\ 3\ 4\ 5)(1\ 2\ 3)(4\ 5)$ ☐

(c) $(1\ 2)(1\ 3)(1\ 4)(2\ 5)$ ☐

(d) $(1\ 2)\ (1\ 3)(1\ 4)$ ☐

UNIT—III

5. The order of element 3 in the group $\{\overline{0}, \overline{1}, \overline{2}, \overline{3}, \overline{4}, \overline{5}\}$; the composition being addition modulo 6 is

(a) 1 ☐

(b) 2 ☐

(c) 3 ☐

(d) 4 ☐

6. What is the remainder if 8^{103} is divided by 103?

(a) 2 ☐

(b) 4 ☐

(c) 8 ☐

(d) 16 ☐

UNIT—IV

7. When $4x^5 \quad 3x^3 \quad 6x^2 \quad 5$ is divided by $2x \quad 1$, the remainder is

(a) 6 ☐

(b) 32 ☐

(c) 0 ☐

(d) 7 ☐

8. The polynomial $x^4 \quad 3x^3 \quad 2x^2 \quad 2x \quad 4$ is divisible by

(a) $x \quad 2$ ☐

(b) $x \quad 2$ ☐

(c) $x \quad 1$ ☐

(d) $x \quad 1$ ☐

UNIT—V

9. The equation $x^7 \quad 3x^4 \quad 2x^3 \quad 1 \quad 0$ has

(a) three positive, two negative and two imaginary roots ☐

(b) three real roots and four imaginary roots ☐

(c) four real roots and three imaginary roots ☐

(d) None of the above ☐

10. If 1 is a multiple root of $x^4 \quad 5x^3 \quad 9x^2 \quad 7x \quad 2 \quad 0$, then it is of multiplicity

(a) 1 ☐

(b) 2 ☐

(c) 3 ☐

(d) 4 ☐

SECTION—B

(Very short questions)

(*Marks* : 15)

Each question carries 3 marks

Answer **one** question from each Unit

UNIT—I

1. Check whether the inverse of matrix

$$\begin{matrix} 3 & 2 & 1 \\ 1 & 1 & 1 \\ 5 & 1 & 1 \end{matrix}$$

exists or not. If exists, find it.

2. Prove that the rank of a matrix A is same as the rank of its transpose.

UNIT—II

3. In a group $(G, \circ)$, show that $(a \circ b)^{\ 1} \quad b^{\ 1} \circ a^{\ 1}$.

4. Prove that the set $G \quad \{1, 2, 3, 4, 5, 6\}$ is a finite Abelian group of order 6 with respect to multiplication modulo 7.

UNIT—III

5. Write the permutation (1 2 3) (4 5) (1 6 7 8 9) (1 5) as the product of disjoint cycles.

6. If n is the order of a and p is prime to n, then show that a^p is also of order n.

UNIT—IV

7. If a polynomial $f(x)$ of degree 2 is divided by $(x \quad)^2$, then prove that the remainder is $(x \quad) f \, (\;) \quad f(\;)$.

8. If $x^2 \quad px \quad 1$ is a factor of $ax^3 \quad bx \quad c$, then find the value of $a^2 \quad c^2$.

UNIT—V

9. Prove that the equation of third degree with real coefficient whose roots are 2 and i is $2x^3 \quad 2x^2 \quad x \quad 2 \quad 0$.

10. Test for the irreducibility of $x^2 \quad 3x \quad 4$ over $\mathbb{Z}$.

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