

2016

(4th Semester)

MATHEMATICS

Paper : MATH-241

(Vector Calculus and Solid Geometry)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(Marks : 50)

The figures in the margin indicate full marks
for the questions

Answer **one** question from each Unit

UNIT—I

1. (a) Prove by vector method

$$\sin(\theta + \phi) = \sin\theta \cos\phi + \cos\theta \sin\phi$$

for any two angles θ and ϕ . 3

- (b) Find a vector whose magnitude is 3 units and which is perpendicular to each of the vectors $\vec{a} = 3\hat{i} + \hat{j} + 4\hat{k}$ and $\vec{b} = 6\hat{i} + 5\hat{j} + 2\hat{k}$. 4

- (c) Find the area of the parallelogram whose diagonals are represented by the vectors $\vec{d}_1 = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{d}_2 = \hat{i} + 3\hat{j} + 4\hat{k}$. 3

2. (a) Prove that

$$[(\vec{a} \times \vec{b})(\vec{b} \times \vec{c})(\vec{c} \times \vec{a})] = 2[\vec{a} \vec{b} \vec{c}]$$
 3

- (b) Prove that a vector function $\vec{f}(t)$ to be a constant magnitude if and only if

$$\vec{f} \cdot \frac{d\vec{f}}{dt} = 0$$
 4

- (c) A particle moves along the curve $x = 2t^2$, $y = t^2 + 4t$, $z = 3t + 5$, where t is the time. Find the velocity and acceleration at time $t = 1$. 3

UNIT—II

3. (a) Find the unit outward drawn normal to the surface $(x - 1)^2 + y^2 + (z - 2)^2 = 9$ at the point $(3, 1, -4)$. 3

- (b) Let $(x, y, z) = x^3 + y^3 + z^3$. Find the directional derivative of u at $(1, -1, 2)$ in the direction of the vectors $\hat{i} + 2\hat{j} + \hat{k}$. 3

(3)

- (c) Suppose $\vec{A} \cdot \vec{0}$. Evaluate $(\vec{A} \cdot \vec{r})$ where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $\vec{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$. 4

4. (a) Show that

$$\oint_S \vec{F} \cdot \vec{n} dS = \int_V \vec{F} \cdot d\vec{v}$$

where $\vec{F} = 4xz\hat{i} + y^2\hat{j} + yz\hat{k}$ and S is the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$. 5

- (b) Find the work done in moving a particle in the force field

$$\vec{F} = 3x^2\hat{i} + (2xz - y)\hat{j} + z\hat{k}$$

along (i) a straight line from (0, 0, 0) to (2, 1, 3) and (ii) the curve defined by $x^2 = 4y, 3x^2 = 8z$ from $x = 0$ to $x = 2$. 5

UNIT—III

5. (a) If by rotation of the axes about the origin, the expression $ax^2 + 2hxy + by^2$ changes to $a'x'^2 + 2h'x'y' + b'y'^2$, then show that $a' = a \cos^2 \theta - b \sin^2 \theta$ and $ab - h^2 = a'b' - h'^2$. 4

(4)

- (b) Prove that the equation

$$2x^2 - 5xy + 3y^2 - 2x - 3y = 0$$

represents a pair of straight lines. Find the coordinates of their point of intersection and the angle between them. 2+2+2=6

6. (a) Reduce the equation

$$14x^2 - 4xy + 11y^2 - 44x - 58y - 71 = 0$$

to the standard form. 5

- (b) Find the equation of the chord of contact of tangents from a given point (x, y) to the conic

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$
 5

UNIT—IV

7. (a) Find the equation of the plane which passes through the point (2, 1, 4) and is perpendicular to the planes

$$9x - 7y + 6z - 48 = 0$$

and $x + y + z = 0$. 5

- (b) If a plane cuts the axes at A, B, C and the centroid of the $\triangle ABC$ is (a, b, c) , then prove that the equation of the plane is

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 3$$
 5

8. (a) Prove that the lines

$$\frac{x-1}{2}, \frac{y-2}{3}, \frac{z-3}{4}$$

and $4x-3y-1=0$, $5x-3z-2=0$ are coplanar. 5

- (b) Find the length and equation of the line of the shortest distance between the lines

$$\frac{x-3}{4}, \frac{y-6}{3}, \frac{z}{2} \text{ and } \frac{x-2}{4}, \frac{y}{1}, \frac{z-7}{1}$$
 5

UNIT—V

9. (a) Find the centre and radius of the sphere given by

$$x^2 + y^2 + z^2 - 4x - 5y - 6z - 1 = 0$$
 5

- (b) Find the equation of the sphere which passes through the origin and touches the sphere $x^2 + y^2 + z^2 = 56$ at the point (2, 4, 6). 5

10. (a) Find the equation of the line of intersection of the plane $3x - 4y - z = 0$ and the cone $15x^2 - 32y^2 - 7z^2 = 0$. 5

- (b) Find the equation of a cylinder whose generating lines have the direction cosines l, m, n and which passes through the circle $x^2 + z^2 = a^2, y = 0$. 5

Subject Code : MATH/IV/04

Booklet No. A

Date Stamp

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To be filled in by the Candidate

DEGREE 4th Semester
(Arts / Science / Commerce /
.....) Exam., **2016**

Subject

Paper

INSTRUCTIONS TO CANDIDATES

- 1. The Booklet No. of this script should be quoted in the answer script meant for descriptive type questions and vice versa.**
- 2. This paper should be ANSWERED FIRST and submitted within 1 (one) Hour of the commencement of the Examination.**
- 3. While answering the questions of this booklet, any cutting, erasing, over-writing or furnishing more than one answer is prohibited. Any rough work, if required, should be done only on the main Answer Book. Instructions given in each question should be followed for answering that question only.**

To be filled in by the Candidate

DEGREE 4th Semester
(Arts / Science / Commerce /
.....) Exam., **2016**

Roll No.

Regn. No.

Subject

Paper

Descriptive Type

Booklet No. B

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Examiner(s)*

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Invigilator(s)*

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MATH/IV/04

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(4th Semester)

MATHEMATICS

Paper : MATH-241

(Vector Calculus and Solid Geometry)

(PART : A—OBJECTIVE)

(Marks : 25)

Answer **all** questions

SECTION—A

(Marks : 10)

Each question carries 1 mark

Put a Tick ☒ mark against the correct answer in the box provided :

1. The value of $[\hat{i} \hat{k} \hat{j}]$ is

(a) 1 ☐

(b) -1 ☐

(c) 0 ☐

(d) None of the above ☐

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(2)

2. The projection of $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ is

(a) $\frac{5}{6}(2\hat{i} + \hat{j} + \hat{k})$ ☐

(b) $\frac{6}{5}(2\hat{i} + \hat{j} + \hat{k})$ ☐

(c) $\frac{6}{5}(\hat{i} + 2\hat{j} + \hat{k})$ ☐

(d) $\frac{5}{6}(\hat{i} + 2\hat{j} + \hat{k})$ ☐

3. If $\vec{a}$ is a constant function, then $\vec{a}$ is

(a) both solenoidal and irrotational ☐

(b) solenoidal ☐

(c) irrotational ☐

(d) neither solenoidal nor irrotational ☐

4. If $\vec{a}$ is any vector and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then $(\vec{a} \cdot \vec{r})\vec{r}$ is

(a) $\vec{a}$ ☐

(b) $\vec{r}$ ☐

(c) $\vec{a} \cdot \vec{r}$ ☐

(d) None of the above ☐

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(3)

5. The equation of the circle $(x - h)^2 + (y - k)^2 = r^2$, when the origin is transferred to the point (h, k) is

(a) $x^2 + y^2 = r^2$ ☐

(b) $x^2 + y^2 = r^2$ ☐

(c) $(x - h)^2 + y^2 = r^2$ ☐

(d) $x^2 + (y - k)^2 = r^2$ ☐

6. If m_1 and m_2 are the slopes of the two lines that the equation $ax^2 + 2hxy + by^2 = 0$ represents, then m_1 and m_2 are connected by the relation

(a) $m_1 + m_2 = \frac{2h}{a}$ ☐

(b) $m_1 m_2 = \frac{b}{a}$ ☐

(c) $m_1 - m_2 = \frac{2h}{a}$ ☐

(d) $m_1 m_2 = \frac{h}{a}$ ☐

7. The equation of the plane through the point $(2, 3, 5)$ and parallel to the plane $2x + 4y + 3z = 9$ is

(a) $2x + 4y + 3z = 7$ ☐

(b) $2x + 4y + 3z = 7$ ☐

(c) $2x + 4y + 3z = 9$ ☐

(d) $2x + 4y + 3z = 9$ ☐

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8. The coordinates of the point of intersection of the line

$$\frac{x-1}{1} = \frac{y-3}{3} = \frac{z-2}{2}$$

with the plane $3x - 4y - 5z = 5$ are

- (a) $(r-1, 3r-3, 2r-2)$ ☐
- (b) $(r-1, 3r-3, 2r-2)$ ☐
- (c) $(r-1, -3r-3, 2r-2)$ ☐
- (d) $(r-1, 3r-3, -2r-2)$ ☐
9. The equation of a sphere which passes through the origin and makes equal intercepts of unit length of the axes is
- (a) $(x-1)^2 + (y-1)^2 + (z-1)^2 = 0$ ☐
- (b) $x^2 + y^2 + z^2 = r^2$ ☐
- (c) $(x-1)^2 + (y-1)^2 + (z-1)^2 = 0$ ☐
- (d) $x^2 + y^2 + z^2 = 1$ ☐
10. The general equation of a curve which touches the three coordinate planes is
- (a) $fx + gy + hz = 0$ ☐
- (b) $\sqrt{fx} + \sqrt{gy} + \sqrt{hz} = 0$ ☐
- (c) $fgh + xyz = 0$ ☐
- (d) $fy + gx + hz = 0$ ☐

where f, g, h are parameters and carry their usual meaning.

(5)

SECTION—B

(Marks : 15)

Each question carries 3 marks

State *True* or *False* by putting a Tick ☒ mark in the box provided and give a brief justification :

1. The vector of magnitude 6 which is perpendicular to both the vectors $\vec{a} = 4\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} + 2\hat{k}$ is $(\hat{i} + 2\hat{j} + 2\hat{k})$.

True ☐ *False* ☐

Justification :

(6)

2. $(\vec{F}) = 0.$

True ☐ False ☐

Justification :

(7)

3. The equation of the diameter of the conic $4x^2 - 6xy + 5y^2 - 1$ conjugates to the diameter $y = 2x$ is $10x - 7y = 0$.

True ☐ False ☐

Justification :

(8)

4. The condition that the line

$$\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$$

is parallel to the plane $ax + by + cz + d = 0$ is $al + bm + cn = 0$.

True ☐ False ☐

Justification :

(9)

5. The equation

$$4x^2 - y^2 - 2z^2 - 2xy - 3yz - 12x - 11y - 6z - 4 = 0$$

represents a cone with vertex (1, 2, 3).

True ☐ False ☐

Justification :
