2016

(6th Semester)

MATHEMATICS

Paper : Math–363

(Mechanics)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **five** questions taking **one** from each Unit

UNIT—I

1. *(a)* A heavy uniform rod rests in a limiting equilibrium within a fixed rough hollow sphere. If   be the angle of friction and 2   the angle subtended by the rod at the centre of the sphere, then show that the inclination   of the rod to the horizon is given by

$$2\tan \quad \tan(\quad) \quad \tan(\quad)$$ 5

(b) The moment of a system of coplanar forces (not in equilibrium) about three collinear points A, B, C in the plane are G_1, G_2, G_3. Prove that

$$G_1 \quad BC \quad G_2 \quad CA \quad G_3 \quad AB \quad 0$$ 5

2. *(a)* If a system of forces in one plane reduces to a couple whose moment is G and when each force is turned round its point of application through a right angle it reduces to a couple of moment H, then prove that when each force is turned through an angle , the system is equivalent to a couple whose moment is $G\cos \quad H\sin$. For what values of   will the moment of the new couple be equal to the moment of the old couple? 5+2=7

(b) How high can a particle rest inside a rough hollow sphere of radius a, if the coefficient of friction be $\frac{1}{\sqrt{3}}$? 3

UNIT—II

3. *(a)* State and prove parallel axes theorems on moments of inertia. 5

(b) Let AB and AC are two uniform rods of length $2a$ and $2b$ respectively. If BAC , then prove that the distance from A of the centre of gravity of the two rods is

$$\frac{(a^4 \quad 2a^2b^2\cos \quad b^4)^{1/2}}{a \quad b}$$ 5

4. *(a)* A thin uniform wire is bent into a ABC. Prove that its centre of gravity is the same as that of the three weights

$$\frac{b \quad c}{2}, \frac{c \quad a}{2}, \frac{a \quad b}{2}$$

placed at A, B, C respectively. Where a, b and c are the lengths of the sides BC, CA and AB. 5

(b) A uniform circular lamina of radius $3a$ and centre O has a hole in the form of equilateral triangle of side $2a$ with one vertex at O. Prove that the centre of gravity from O is $\frac{2a}{9 \quad \sqrt{3}}$. 5

UNIT—III

5. *(a)* For a particle moving in a plane curve, show that the tangential and normal components of accelerations are

$$v\frac{dv}{ds} \text{ and } \frac{v^2}{\ }$$

where is the radius of curvature. 5

(b) A particle P moves in the curve $y \quad a \log \sec \frac{x}{a}$ in such a way that the tangent to the curve at P rotates uniformly. Prove that the resultant acceleration of the particle varies as the square of the radius of curvature. 5

6. *(a)* For $\frac{1}{m}$ of the distance between two stations, a train is uniformly accelerated and for $\frac{1}{n}$ of the distance, it is uniformly retarded. It starts from rest at one station and comes to rest at the other. Prove that the ratio of its greatest velocity to its average velocity is

$$1 \quad \frac{1}{m} \quad \frac{1}{n} : 1$$ 5

(b) A particle is moving with SHM and while making an excursion from one position of rest to the other, its distances from the middle point of its path at three consecutive seconds are observed to be x_1, x_2, x_3. Prove that the time of complete oscillation is

$$\frac{2}{\cos^{\ 1} \frac{x_1 \quad x_3}{2x_2}}$$ 5

UNIT—IV

7. *(a)* A projectile aimed at mark which in the horizontal plane through the point of projection, falls a metre short of it when the elevation is and goes b metre far when the elevation is . Show that if the velocity of projection be the same in all the cases, the proper elevation is

$$\frac{1}{2}\sin^{1}\frac{a\sin 2 \qquad b\sin 2}{a \quad b}$$ 5

(b) A particle is projected from a point on the ground level and its height is h when it is at a horizontal distance a and $2a$ from its point of projection. Prove that the velocity of projection u is given by

$$u^2 \quad \frac{g}{4}\ \frac{4a^2}{h} \quad 9h$$ 5

8. (a) If r and r are the maximum ranges up and down the inclined plane respectively, prove that

$$\frac{1}{r} \quad \frac{1}{r}$$

is independent of the inclination of the plane. Also show that

$$\frac{1}{r} \quad \frac{1}{r} \quad \frac{2}{R}$$

where R is the maximum range on a horizontal plane with the same velocity of projection. 5

(b) A particle, projected vertically upwards with a velocity U in a medium whose resistance varies as the square of the velocity, will return to the point of projection with velocity $v \quad \frac{UV}{\sqrt{U^2 \quad V^2}}$

after a time

$$\frac{V}{g} \quad \tan^{1}\frac{U}{V} \quad \tanh^{1}\frac{v}{V}$$

where V is the terminal velocity. Prove it. 5

UNIT—V

9. (a) Deduce the work-energy equation. 5

(b) A uniform elastic string has the length a_1 when the tension is T_1 and the length a_2 when the tension is T_2. Show that its natural length is

$$\frac{a_2 T_1 \quad a_1 T_2}{T_1 \quad T_2}$$

and the amount of work done in stretching it from its natural length to a length $a_1 \quad a_2$ is

$$\frac{1}{2}\frac{(a_1 T_1 \quad a_2 T_2)^2}{(T_1 \quad T_2)(a_1 \quad a_2)}$$ 5

10. *(a)* Two spheres of masses M, m impinge directly when moving in opposite directions with velocities u and v respectively. If the sphere of mass m is brought to rest by the collision, show that

$$v(m \quad eM) \quad M(1 \quad e)u$$ 5

(b) A smooth billiard ball impinges obliquely on another equal ball at rest in a direction making an angle   with the line of centres at the moment of impact. If the coefficient of restitution of the two balls is $\frac{1}{3}$, prove that the angle through which the direction of motion of the impinging ball deviates is

$$\tan^{1} \frac{2\tan}{1 \quad 3\tan^{2}}$$ 5

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Subject Code : MATH/VI/11

Booklet No. A

Date Stamp ....

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To be filled in by the Candidate

DEGREE 6th Semester
(Arts / Science / Commerce / ....) Exam., **2016**

Subject ....

Paper ....

INSTRUCTIONS TO CANDIDATES

1. **The Booklet No. of this script should be quoted in the answer script meant for descriptive type questions and vice versa.**
2. **This paper should be ANSWERED FIRST and submitted within 1 (one) Hour of the commencement of the Examination.**
3. **While answering the questions of this booklet, any cutting, erasing, over-writing or furnishing more than one answer is prohibited. Any rough work, if required, should be done only on the main Answer Book. Instructions given in each question should be followed for answering that question only.**

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DEGREE 6th Semester
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Roll No. ....

Regn. No. ....

Subject ....

Paper ....

Descriptive Type
Booklet No. B ....

Signature of Scrutiniser(s)

Signature of Examiner(s)

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(6th Semester)

MATHEMATICS

Paper : Math–363

(Mechanics)

(PART : A—OBJECTIVE)

(*Marks* : 25)

The figures in the margin indicate full marks for the questions

SECTION—A

(*Marks* : 10)

Put a Tick ☑ mark against the correct answer in the box provided : 1×10=10

1. The least force P required to pull a body up on an inclined plane inclined at an angle to the horizontal is

(a) P $W \sin(\quad)$ ☐

(b) P $W \sin(\quad)$ ☐

(c) P $W \cos(\quad)$ ☐

(d) P $W \cos(\quad)$ ☐

2. A system of forces in a plane is in equilibrium, if the algebraic sums of the

(a) resolved parts in any two parallel directions vanish ☐

(b) moments of all the forces w.r.t. each of three collinear points are zero ☐

(c) moments about each of the two given points not vanish ☐

(d) resolved parts in any two perpendicular directions vanish ☐

3. Let $(\bar{x}, \bar{y}, \bar{z})$ be the centre of mass of a system of particles with respect to the x, y and z-axes and let parallel x , y , z axes be taken through $(\bar{x}, \bar{y}, \bar{z})$. If F and F be the products of inertia with respect to the x, y axes and with respect to x , y axes, then

(a) $F \quad M\bar{x}\bar{z} \quad F$ ☐

(b) $F \quad M\bar{y}^2 \quad F$ ☐

(c) $F \quad M\bar{x}\bar{y} \quad F$ ☐

(d) $F \quad M\bar{z}^2 \quad F$ ☐

4. The CG of a uniform semi-circular lamina of radius r lies on the axis of its symmetry at a distance from the centre

(a) $\frac{2r}{}$ ☐

(b) $\frac{2\sqrt{2}r}{}$ ☐

(c) $\frac{}{2\sqrt{2}r}$ ☐

(d) $\frac{}{2r}$ ☐

5. If the position of a moving particle at time t referred to rectangular axes is given by $x \quad at$, $y \quad bt \quad ct^2$, where a, b and c are constants, then its acceleration at time t is

(a) a along the x-axis ☐

(b) $a \quad b \quad c$ along the x-axis ☐

(c) $2c$ along y-axis ☐

(d) $\sqrt{a^2 \quad b^2}$ along the y-axis ☐

6. The equation of SHM of period T of a particle is

(a) $x \qquad T^2 x$ ☐

(b) $x \qquad \frac{4\;\;^{2}}{T^{2}}x$ ☐

(c) $x \qquad \frac{4T^{2}}{\;\;2}x$ ☐

(d) $x \qquad \frac{1}{T^{2}}x$ ☐

7. The least velocity with which a body can be projected to have a horizontal range R is

(a) $\sqrt{gR}$ ☐

(b) $\sqrt{g / R}$ ☐

(c) $\sqrt{R / g}$ ☐

(d) $R\sqrt{g}$ ☐

8. A particle of mass m is let fall from a height h in a medium whose resistance is mk (velocity)^2. The terminal velocity of the particle is given by

(a) $\sqrt{h / g}$ ☐

(b) $\sqrt{g / h}$ ☐

(c) $\sqrt{k / g}$ ☐

(d) $\sqrt{g / k}$ ☐

9. A smooth ball falling vertically from a height x impinges on a horizontal fixed plane. If e is the coefficient of restitution, then the ball rebounds to a height

(a) e^2x ☐

(b) ex ☐

(c) e / x ☐

(d) x / e ☐

10. A smooth sphere impinges directly with velocity u on another smooth sphere of equal mass at rest. If the spheres are perfectly elastic, then the velocity of second sphere after collision will be

(a) 0 ☐

(b) u ☐

(c) $\frac{u}{2}$ ☐

(d) None of the above ☐

SECTION—B

(*Marks* : 15)

Answer the following questions : 3×5=15

1. Forces proportional to 1, 2, 3 and 4 act along the sides AB, BC, AD and DC respectively of a square $ABCD$ the length of whose sides is 2 ft. Find the magnitude and the line of action of their resultant.

2. Find the centre of gravity of a sectoral area of a circle bounded by the curve r f ( ) and the radius vector and .

3. The maximum velocity of a body moving with SHM is 2 cm/sec and its period is 1/5 sec. What is its amplitude?

4. If h and h be the greatest heights in the two paths of a projectile with a given velocity for a given range R, then show that

$$R \quad 4\sqrt{hh}$$

5. A ball of mass 4 kg moving with a velocity 100 cm/sec overtakes a ball of mass 6 kg moving with velocity 50 cm/sec in the same direction. If $e \quad \frac{1}{2}$, then find the velocities of the balls after impact.

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