GOVERNMENT ZIRTIRI RESIDENTIAL SCIENCE COLLEGE

Subject : Mathematics
Paper name : Advanced Calculus
Paper No : MAT 362
Semester : 6th semester

A. Multiple choice questions [25 (5 from each unit)]

1. The Upper Darbaux sums of a function *f* corresponding to the partition *P* of interval [a,b] is given by the relation]
   a) $L(P, f) = \sum_{i=2}^{n} m_i \Delta x_i$
   b) $L(P, f) = \sum_{i=2}^{n} M_i \Delta x_i$
   c) $U(P, f) = \sum_{i=2}^{n} m_i \Delta x_i$
   d) $U(P, f) = \sum_{i=2}^{n} M_i \Delta x_i$

2. The Lower Riemann integral for a function *f* corresponding to the partition *P* of interval is given by the relation
   a) $\sup\ L(P, f) = \int_{\underline{a}}^{b} f\ dx$
   b) $L(P, f) = \sup \int_{a}^{\bar{b}} f\ dx$
   c) $L(P, f) = \inf \int_{\underline{a}}^{b} f\ dx$
   d) $\sup\ U(P, f) = \int_{\underline{a}}^{b} f\ dx$

3. Let P^* be a refinement of a partition *P,* then for a bounded function *f*
   a) $L(P^*, f) \le L(P, f)$
   b) $L(P^*, f) \ge L(P, f)$
   c) $U(P^*, f) \ge L(P, f)$
   d) $U(P^*, f) \le U(P, f)$

4. For any two partitions P_1, P_2 of a bounded function *f*
   a) $L(P_1, f) \le U(P_2, f)$
   b) $L(P_1, f) < U(P_2, f)$
   c) $L(P_2, f) \le U(P_1, f)$
   d) $U(P_2, f) \le L(P_1, f)$

5. If a bounded function f is integrable on [a,b], then

   a) $\lim\limits_{\mu(P)\to\infty} S(P,f)=\int\limits_a^b f\,dx$

   b) $\lim\limits_{\mu(P)\to 0} S(P,f)=\int\limits_a^b f\,dx$

   c) $\int_{\underline{a}}^{b} f\,dx=\int_a^{\overline{b}} f\,dx$

   d) $L(P,f)=U(P,f)=S(P,f)$

   where $L(P,f)$, $U(P,f)$ and $S(P,f)$ are the lower Darboux, upper Darboux and Riemann sum of f corresponding to a partition P of [a,b] with norm $\mu(P)$.

6. If f and g be two positive functions such that $f(x)\le g(x)\ \forall \in[a,b]$, then

   a) $\int_a^b g\,dx$ converges if $\int_a^b f\,dx$ converges

   b) $\int_a^b f\,dx$ converges if $\int_a^b g\,dx$ converges

   c) $\int_a^b f\,dx$ diverges if $\int_a^b g\,dx$ diverges

   d) $\int_a^b f\,dx$ and $\int_a^b g\,dx$ behaves alike

7. The improper integral $\int_a^\infty \frac{dx}{x^n}$, a > 0 converges if and only if

   a) $n\le 1$
   b) $n<1$
   c) $n>1$
   d) $n\ge 1$

8. The improper integral $\int\limits_a^b \frac{dx}{(x-a)^n}$ converges if and only if

   a) $n\le 1$
   b) $n<1$
   c) $n>1$
   d) $n\ge 1$

9. If f and g be two positive functions on $[a,b]$ such that

   $\lim\limits_{x\to a^+} \frac{f(x)}{g(x)}=l,$ a non-zero finite number , then

   a) $\int_a^b g\,dx$ converges if $\int_a^b f\,dx$ converges

   b) $\int_a^b f\,dx$ converges if $\int_a^b g\,dx$ converges

c) $\int_a^b f\,dx$ diverges if $\int_a^b g\,dx$ diverges

d) $\int_a^b f\,dx$ and $\int_a^b g\,dx$ behaves alike

10. Which of the following definite integral is an improper integral?

a) $\int_0^{\frac{\pi}{2}} \sin x dx$

b) $\int_{-1}^{1} \frac{dx}{1+x^2}$

c) $\int_0^4 \frac{dx}{(x-2)(x-3)}$

d) $\int_0^1 \frac{dx}{x(1+x)}$

11. Uniformly convergent improper integral of a continuous function is

a) not continuous

b) itself continuous

c) may be continuous

d) differentiable

12. The value of the improper integral $\int_0^\infty e^{-x^2} dx$ is equal to

a) $\frac{\pi}{2}$

b) $\frac{2}{\pi}$

c) $\frac{\sqrt{\pi}}{2}$

d) $\frac{2}{\sqrt{\pi}}$

13. The value of the improper integral $\int_0^\infty e^{-x^2} \cos\alpha x\, dx$ is equal to

a) $\frac{\sqrt{\pi}}{2}$

b) $\frac{\sqrt{\pi}}{2} e^{-\frac{\alpha}{4}}$

c) $\frac{\sqrt{\pi}}{2}e^{-\frac{\alpha^2}{4}}$

d) $\frac{\pi}{\sqrt{2}}e^{-\frac{\alpha^2}{4}}$

14. The improper integral $\int_0^\infty x^{n-1}e^{-x}dx$ is convergent if and only if

a) $n \le 1$
b) $n < 1$
c) $n > 1$
d) $n > 0$

15. The value of the improper integral $\int_0^\pi \frac{dx}{a+b\cos x}$ if *a* is positive and $|b| < a$ is

a) $\frac{2\pi}{(a^2-b^2)^{1/2}}$

b) $\frac{2\pi}{(a^2-b^2)^{1/2}}$

c) $\frac{\pi}{(a^2-b^2)^{1/2}}$

d) $\frac{\pi}{(a^2-b^2)^{3/2}}$

16. The value of the double integral $\int_{1/2}^{1}\int_{1/2}^{1} \frac{x-y}{x+y}dxdy$ is

a) 0

b) $-\frac{1}{2}$

c) $\frac{1}{2}$

d) $\frac{\pi}{2}$

17. The value of the integral $\int_C x^2dx + xydy$ taken along the line segment from (1,0) to (0,1) is

a) 0

b) $-\frac{1}{6}$

c) 1

d) $\frac{1}{6}$

18. The value of the integral $\int_C xy\,dx$ along the arc of the parabola $x = y^2$ from (1, -1) to (1, 1) is

a) 0
b) $\frac{2}{5}$
c) $\frac{4}{5}$
d) $\frac{5}{4}$

19. The value of the double integral $\iint x^2 y^3 dxdy$ over the circle $x^2 + y^2 = a^2$ is

a) 0
b) $-\frac{1}{2}$
c) $\frac{1}{2}$
d) $\frac{\pi}{2}$

20. The value of the integral $\int_C \frac{dx}{x+y}$ where C is the curve $x = at^2,\ y = 2at,\ \ 0 \le t \le 2$ is

a) $\frac{1}{6}$
b) $\log 4$
c) $-\frac{1}{6}$
d) $\log 2$

21. By M_n- test , the sequence $\{f_n\}$ converges uniformly to f on $[a,b]$ if and only if
a) $M_n \to 0$ as $n \to 0$
b) $M_n \to \infty$ as $n \to 0$
c) $M_n = \inf\{|f_n(x) - f(x)| : x \in [a,b]\}$
d) $M_n \to 0$ as $n \to \infty$

22. With regards to uniform and point-wise convergence of sequences in $[a,b]$, which of the following is/are true?
a) point-wise convergence $\Rightarrow$ uniform convergence
b) uniform convergence $\Rightarrow$ point-wise convergence

c) uniform limit = point-wise limit
d) none of the above

23. The sequence of function given by $f_n(x) = \dfrac{n}{x+n}$ is

a) uniformly convergent in [0, k], whatever k may be
b) onlypointwise convergent in [0, k], whatever k may be
c) not convergent at all in [0, k], whatever k may be
d) uniformly convergent in $[0, \infty)$

24. The sequence of function given by $f_n(x) = \dfrac{nx}{1+n^2x^2}$ is

a) uniformly convergent in any interval containing zero
b) notuniformly convergent in any interval containing zero
c) pointwise convergent with pointwise limit 1
d) none of the above

25. The sequence of function $f_n(x) = nxe^{-nx^2}$ is point-wise, but not uniformly convergent on

a) [0, k], where k< 1
b) $[0, \infty]$
c) $[0, \infty)$
d) $(0, \infty)$

B. Fill up the blanks [15 (3 from each unit)]

Unit I

1. The function defined by $f(x) = \begin{cases} 0, \text{when x is rational} \\ 1, \text{when x is irrarional} \end{cases}$ is not ________ on any interval.
2. If f_1 and f_2 are two bounded and integrable functions on [a,b], then ______is also integrable on[a,b].
3. Every continuous and constant function is ______

Unit 2

4. The improper integral (beta function) $\int_0^1 x^{m-1}(1-x)^{n-1}dx$ exists if and only if *m,n* are both ______.
5. By Frulanni's integral,if ϕ is continuous in $[0, \infty)$ and $\lim_{x \to 0} \phi(x) = \phi_0$ ______ then we have $\int_0^\infty \frac{\phi(ax) - \phi(bx)}{x} dx = (\phi_0 - \phi_1) \log\left(\frac{b}{a}\right)$.
6. Every ________ integral is convergent.

Unit 3

7. Let $\phi(y)=\int_a^b f(x,y)dx$, if $f(x,y)$ is continuous and f_y also exists in [a,b;c,d], then ϕ is ______

8. The convergent improper integral $\int_0^\infty e^{-x^2}\cos yx\,dx$ in $(-\infty,\infty)$ is ______

9. Let $f(x,y)$ be a continuous function of two variable with rectangle $[a,b;c,d]\subseteq R^2$,

   then the function defined $\phi(y)=\int_a^b f(x,y)dx$ by is ______ in [c,d].

Unit 4

10. The value of the double integral $\iint \frac{x-y}{x+y}dxdy$ over $\left[\frac{1}{2},1;\ \frac{1}{2},1\right]$ is ______.
11. The value of $\int_C 4x^3 ds$ where C is the line segment from (-2,-1) to (1, 2) is ______
12. A domain E will is said to be regular or quadratic with respect to y-axis, if it is _______ by curves of the form $y=\phi(x),\ y=\varphi(x);\ \ x=a,\ x=b$ where ϕ and φ are continuous and $\phi(x)<\varphi(x)\ \forall x\in[a,b]$

Unit 5

13. The sequence $f_n(x)=x^n$ is uniformly convergent on $[0,k]$ where k is a number less than ______
14. If a sequence $\{f_n\}$ converges uniformly to f on $x\in[a,b]$ and let f_n be integrable $\forall\, n$, then f is integrable and $\lim_{n\to\infty}\int_a^x f_n(x)dx=$______.
15. The sequence $\{f_n\}$ of continuous function is uniformly convergent on [a,b] to a function *f*. Then f is also ______

Key Answers

A. Multiple choice questions [replace x]

1. d)	2. a)	3. d)	4. a)	5. b)	6. b)	7. c)
8. b)	9. d)	10. c)	11. b)	12. c)	13. c)	14. d)
15. c)	16. a)	17. b)	18. c)	19. a)	20. b)	21. d)
22. b)	23. a)	24. b)	25. c)			

B. Fill up the blanks [replace x]

1. integrable
2. f_1+f_2

3. Riemann integrable
4. positive
5. $\lim_{x \to \infty} \phi(x) = \phi_1$
6. absolutely convergent
7. derivable
8. uniformly convergent
9. continuous
10. 0
11. $-15\sqrt{2}$
12. bounded
13. 1
14. $\int_a^x f(x)dx$
15. continuous