GOVERNMENT ZIRTIRI RESIDENTIAL SCIENCE COLLEGE

Subject : Mathematics
Paper name : Modern Algebra
Paper no : Math 361
Semester : 6^{th}

A. Multiple Choice Question

1. The necessary and sufficient condition for a homomorphism f of a group G with identity e into a group G` with kernel K to be an isomorphism of G into G` is that
a) K=ϕ
b) K=(e)
c) K=G
d) K=G`

2. If f is a homomorphism of G into G` then K is the kernel of f if
a) K = {x ϵ G: f(x)=e}
b) K = {x ϵ G: f(x)=0}
c) K = {x ϵ G: f(x)=e`}
d) K = {x ϵ G: f(e)=e`}

3.If G is a group, the mapping f_a: G→G` is an inner automorphism if
a) $f_a(x)=a^{-1}xa$
b) $f_a(x)=ax^{-1}a^{-1}$
c) $f_a(x)= xax^{-1}$
d) $f_a(x)=x^{-1}ax$

4.A subgroup H of a group G is normal if and only if
a) $x^{-1}Hx = H$
b)$xHx^{-1} = x$
c) $xH^{-1}x^{-1} = H$
d) $xHx^{-1} = H$

5.If H is a normal subgroup of a group G and K a normal subgroup of G containing H then
a) G/K ≅ (G/H)/(K/H)
b) G/K ≅ (K/H)/(G/H)
c) G/K ≅ (H/K)/(H/G)
d) G/K ≅ (G/H)

6.The necessary and sufficient condition for a non-empty subset S of a ring R to be a subring are
a) a∈S, b∈S ⇒ a+b ∈ S and ab ∈ S
b) a∈S, b∈S ⇒ a-b ∈ S and ab ∈ S
c) a∈S, b∈S ⇒ a+b ∈ S and a/b ∈ S
d) a∈S, b∈S ⇒ a-b ∈ S and a/b ∈ S

7.In the ring of integers I the maximal ideal is
a) 6
b) 10
c) 8

d) 5

8.The necessary and sufficient condition for a non-empty subset K of a field F to be a subfield of F are
a) a∈K, b∈K ⇒ a+b ∈ K and ab^{-1} $\in K$
b) a∈K, b∈K ⇒ a+b ∈ K and $a^{-1}b$ $\in K$
c) a∈K, b∈K ⇒ a-b ∈ K and ab^{-1} $\in K$
d) a∈K, b∈K ⇒ a-b ∈ K and $a^{-1}b \in K$

9.Which of the following statement is false ?
a) A commutative ring with unity is a field if it has no proper ideal.
b) R is a commutative ring and a∈R then Ra = (ra : r ∈ R) is an ideal of R
c) A field has no proper ideal i.e the only ideals of a field F is F itself and (0)
d) The ring of an integers is not a principal ideal.

10.An ideal S of a ring R is prime if for all a,b ∈ R. Then
a) ab∈S ⇒ a∈S or b∈S
b) ba ∈S ⇒ a∈S or b∈S
c) a^{-1}b∈S ⇒ a^{-1} ∈S or b∈S
d) ab^{-1} ∈S ⇒ a∈S or b^{-1} ∈S

11.Let f be a homomorphism of a ring R into a ring R`, then
a) ker f = {x∈ R : f(x) = 0`}
b) ker f = {x∉ R : f(x) = 0`}
c) ker f = {x∈R : f(x) = 0}
d) ker f = {x∈R : f(x) ≠ 0`}

12.Let a be a non-zero element in the Euclidean ring R , then a is a unit if
a) d(a) ≠ d(1)
b) d(a) = d(1)
c) d(a) > d(1)
d) d(a) < d(1)

13.Let R be a Euclidean ring and a,b be two non-zero elements in R , the b is a unit in R if
a) d(ab) < d(a)
b) d(ab) > d(a)
c) d(ab) = d(a)
d) d(ab) ≠ d(a)

14.In a commutative ring R with unity 1. An element a∈R is a unit in R if there exist an element b∈R such that
a) ab≠ 1
b)a^{-1}b = 1
c) ab = 1
d) a/b = 1

15.Which of the following statement is false?
a) The ring of integers is not an Euclidean domain
b) Every field is an Euclidean domain

c) Every Euclidean domain is a PID
d) Every Euclidean domain possesses unity

16.Which of the following set of vectors is linearly independent in $V_3(R)$?
a) {(1,2,1),(3,1,5),(3,-4,7)}
b) {(2,-3,1),(3,-1,5),(1,-4,3)}
c) {(2,1,2),(8,4,8)}
d) {(-1,2,1),(3,0,-1),(-5,4,3)}

17.Which of the following functions T from R^2 into R^2 is a linear transformation?
a) $T(x_1, x_2) = (1+x_1, x_2)$
b) $T(x_1, x_2) = (x_1^2, x_2)$
c) $T(x_1, x_2) = (x_1-x_2, 0)$
d) $T(x_1, x_2) = (\sin x_1, x_2)$

18. The necessary and sufficient condition for a non-empty subset W of a vector space V(F) to be a subspace of V are
a) $\alpha \in W, \beta \in W, a \in F \Rightarrow \alpha - \beta \in W \text{ and } a\alpha \in W$
b) $\alpha \in W, \beta \in W, a \in F \Rightarrow \alpha + \beta \in W \text{ and } a\alpha \in W$
c) $\alpha \in W, \beta \in W, a \notin F \Rightarrow \alpha - \beta \in W \text{ and } a\alpha \in W$
d) $\alpha \in W, \beta \in W, a \in F \Rightarrow \alpha / \beta \in W \text{ and } a\alpha \in W$

19.If S and T are finite subsets of a vector space V(F), then
a) $S \subseteq T \Rightarrow L(S) \subseteq L(T)$
b) $S \supseteq T \Rightarrow L(S) \supseteq L(T)$
c) $S \cup T \Rightarrow L(S) \cup L(T)$
d) $S \cap T \Rightarrow L(S) \cap L(T)$

20.If F is a field of complex number then the vector (a_1, b_1) and (a_2, b_2) in $V_2(F)$ are linearly dependent if and only if
a) $a_1a_2 - b_1b_2 = 0$
b) $a_1b_2 - b_1a_2 = 0$
c) $a_1b_2 + b_1a_2 = 0$
d) $a_1b_1 + a_2b_2 = 0$

21.If A = $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$, the eigenvalues of A are
a) 1, -1
b) 1, 0
c) i, -i
d) 1, i

22.Let V and W be vector spaces over the field F. Let T be a linear transformation from V into W . If V is finite dimensional then,
a) rank T + nullity T = dim V
b) rank T – nullity T = dim V
c) rank T + dim V = nullity T
d) rank T – dim V = nullity T

23. A necessary and sufficient condition for a square matrix A of order n over a field F to be diagonisable is that
a) A has exactly n linearly independent eigen vectors
b) A has exactly n linearly dependent eigen vectors
c) A has exactly (n+1) linearly independent eigenvectors
d) A has exactly (n+1) linearly dependent eigenvectors

24.Let T: $R^3 \to R^2$ be a linear transformation defined by T(x,y,z) = (y+z, y-z). Find the matrix T with respect to the ordered basis {(1,0,0),(0,1,0),(0,0,1)} and {(1,0), (0,1)}

a) $\begin{matrix} 0 & 0 & 1 \\ 0 & 1 & -1 \end{matrix}$

b) $\begin{matrix} 0 & 1 & 1 \\ 0 & 1 & -1 \end{matrix}$

c) $\begin{matrix} 0 & 1 & -1 \\ 0 & 1 & -1 \end{matrix}$

d) $\begin{matrix} 0 & 1 & 1 \\ 0 & -1 & 1 \end{matrix}$

25. A square matrix A of order n is similar to a square matrix B of order n if
a) B = $P^{-1}AP$, where P is a symmetric matrix
b) B = PAP, where P is skew-symmetric matrix
c) B = $P^{-1}AP$, where P is a singular matrix
d) B = $P^{-1}AP$, where P is non-singular matrix

B. Fill up the blanks
1. The set Z of all self-conjugate elements of a group G is called ___________.
2. If f is a homomorphism of G into itself , then f is called ____________
3. Every homomorphic image of a group G is __________ to some quotient group of G.
4. A finite commutative ring without zero divisors is a___________.
5. An arbitrary intersection of ___________ is a subrings.
6. An ideal generated by a single element of itself is called ___________.
7. If f is a homomorphism of a ring R into a ring R`, then the kernel of f is _________ of R
8. Every homomorphic image of a commutative ring is ____________.
9. Let R be a Euclidean domain and a&b be two non-zero elements of R. Then b is non-unit in R if ______________.
10.The intersection of any two subspaces W_1&W_2 of a vector space V(F) is also a _________ of V(F).
11. The ________ of two subspaces is not necessarily a subspaces.
12. Let W_1,W_2 be two subspaces of a finite dimensional vector space V(F) ,then dim(W_1+W_2) = ______________.
13 If A = $\begin{pmatrix} 0 & 1 \\ -4 & 4 \end{pmatrix}$, the eigenvalues of A are ________.
14. If V is finite dimensional, the rank of T is the __________ of the range of T.
15. Similar matrices have the same ___________ polynomial.

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KEY ANSWER

A. Multiple choice questions

1.b) 2.c) 3.a) 4.d) 5.a) 6.b) 7.d) 8.c) 9.d) 10.a) 11.a) 12.b) 13.c) 14.c) 15.a) 16.b) 17.c) 18.a) 19.a) 20.b) 21.c) 22.a) 23.a) 24.b) 25.d)

B. Fill up the blanks

1. Center of G
2. Endomorphism
3. Isomorphic
4. Field
5. Subring
6. Principal ideal
7. An ideal
8. Commutative
9. d(ab)>d(a)
10. subspace
11. union
12. $\dim W_1 + \dim W_2 - \dim (W_1 \cap W_2)$
13. (2,2)
14. Dimension
15. characteristics