GOVERNMENT ZIRTIRI RESIDENTIAL SCIENCE COLLEGE

Subject : Mathematics

Paper Name : Advanced Calculus

Paper No : MATH/6/CC/362

Semester : 6th semester

A. Multiple Choice Questions[25(5 from each unit)]

1. If $f_1, f_2 \in R[a,b]$ then the odd one is

(a) $f_1^2 \in R[a,b]$

(b) $f_1 + f_2 \in R[a,b]$

(c) $f_1 / f_2 \in R[a,b]$

(d) $f_2^2 \in R[a,b]$

2. If f is defined on $[a,b]$ by $f(x) = k, \forall x \in [a,b]$ then

(a) $f \in R[a,b]$

(b) $f \notin R[a,b]$

(c) $\int_a^b k = k(b-a)$

(d) Both (a) and (c)

3. If f is defined on [0,1] by

$f(x) = 0$, when x is rational

$f(x) = 1$, when x is irrational , then

(a) f is bounded on [0,1]

(b) $f \notin R[a,b]$

(c) Both (a) and (b)

(d) None of the above

4. If f is a function defined on [-1,1] by $f(x) = |x|$ then which one of the following is incorrect

(a) f is bounded

(b) f is continuous

(c) f is integrable

(d) f is not integrable

5. If P^* is a refinement of P, then for a bounded function f

(a) $U(P^*, f) \le U(P, f)$

(b) $U(P^*, f) \ge U(P, f)$

(c) $U(P^*, f) = U(P, f)$

(d) None of the above

6. If f and g are two positive functions such that $f(x) \le g(x), \forall x \in [a,b]$ then the improper integral

(a) $\int_a^b g dx$ converges if $\int_a^b f dx$ diverges

(b) $\int_a^b f dx$ converges if $\int_a^b g dx$ converges

(c) $\int_a^b f dx$ diverges if $\int_a^b g dx$ diverges

(d) Both (b) and (c)

7. The definite integral $\int_1^4 \frac{dx}{(x-1)(4-x)}$ is

(a) Improper integral of first kind

(b) Improper integral of second kind

(c) Improper integral of third kind

(d) None of the above

8. By Frullani's integral $\int_0^\infty \frac{e^{-ax} - e^{-bx}}{x} dx$ equals to

(a) $2\log\frac{b}{a}$

(b) $\log\frac{a}{b}$

(c) $\log\frac{b}{a}$

(d) $\log(a+b)$

9. The integral $\int_1^\infty \frac{dx}{x\sqrt{x^2+1}}$

(a) Converges

(b) Diverges

(c) Both (a) and (b)

(d) None of the above

10. Consider the improper integral

(I) . $\int_0^1 \frac{dx}{\sqrt{1-x}}$ (II). $\int_0^1 \frac{dx}{x^2}$

Then

(a) (I) is convergent but (II) is divergent

(b) (I) is divergent but (II) is convergent

(c) Both (I) and (II) are convergent

(d) Neither (I) nor (II) is convergent

11. The integral $\int_2^\infty \frac{2x^2}{x^4-1}dx$ is

(a) Convergent

(b) Divergent

(c) Neither converge nor diverge

(d) None of the above

12. If the function $f(x,y)$ and $f_n(x,y)$ exist and continuous in $[a,b;c,d]$ then

(a) derivative of $\int_a^b f(x,y)dx$ with respect to y is not possible to determine

(b) derivative of $\int_a^b f(x,y)dx$ with respect to y is always possible to determine

(c) derivative of $\int_a^b f(x,y)dx$ with respect to y is continuous

(d) None of the above

13. The improper integral $\int_0^\infty e^{-x^2}\cos yx dx$ is

(a) uniformly convergent in $(-\infty,\infty)$

(b) not uniformly convergent in $(-\infty,\infty)$

(c) Divergent

(d) None of the above

14. If $f(x,y)=\frac{y^2}{x^2+y^2}$ and $g(y)=\int_0^1 f(x,y)dx$, then

(a) $g'(0-)=g'(0+)$

(b) $g'(0-)\neq g'(0+)$

(c) $g'(0+)=\pi$

(d) None of the above

15. Which of the following is Jordan Curve

(a) Parabola

(b) Hyperbola

(c) Straight line

(d) Ellipse

16. Choose the correct one

(a) $\int_{-C} fdx+gdy=-\int_C fdx+gdy$

(b) $\int_C fdx+gdy=-\int_{-C} fdx-gdy$

(c) $\int_{-C} fdx+gdy=-\int_{-C} fdx+gdy$

(d) $\int_{-C} fdx+gdy=\int_C fdx-gdy$

17. The integral $\int_C x^2dx+xydy$ taken along the line segment from (1,0) to (0,1) equals

(a) $\frac{-1}{6}$

(b) $\frac{1}{6}$

(c) $\frac{1}{2}$

(d) $-\frac{1}{2}$

18. The value of $\iint xy(x+y)dxdy$ over the area between $y-x^2=0$ and $y-x=0$ is

(a) $\frac{3}{56}$

(b) $\frac{56}{3}$

(c) $\frac{3}{46}$

(d) 56

19. The pointwise limit of sequence of real valued function

$$f_n(x)=\sin x+\frac{x}{n}, \forall x \in IR$$

(a) $f(x)=0, \forall x \in IR$

(b) $f(x)=1, \forall x \in IR$

(c) $f(x)=\sin x, \forall x \in IR$

(d) Does not exist

20. The uniform limit of sequence of real valued function

$f_n(x) = x - \frac{x^n}{n}, \forall x \in [0,1]$ is

(a) $f(x) = 0, \forall x$

(b) $f(x) = x, \forall x$

(c) $f(x) = \begin{cases} 0, x = 0 \\ 1, x = 1 \\ x, 0 < x < 1 \end{cases}$

(d) None of the above

21. The sequence $f_n(x) = x^n$ is

(a) Uniformly convergent on $[0,k], k < 1$

(b) Uniformly convergent on $[0,1]$

(c) Not uniformly convergent

(d) None of the above

22. The infinite series $\sum_{n=1}^{\infty} \frac{1}{n^3(1+nx^2)}, \forall x \in IR$

(a) Can be differentiated term by term

(b) Cannot be differentiated term by term

(c) Both (a) and (b)

(d) None of the above

23. The sequence of the function $f_n(x) = \frac{nx}{1+n^2x^2}, x \in IR$ is

(a) Pointwise convergent

(b) Pointwise limit $f(x) = 0, \forall x \in IR$

(c) Not uniformly convergent in any interval $[a,b]$ with 0 as interior point

(d) All of the above

24. The series $\sum_{n=1}^{\infty} \frac{\sin nx}{n^p}, p>1$

(a) Converges uniformly for all real values of x

(b) Does not converge uniformly

(c) Diverges

(d) None of the above

25. Let $f_n(x) = \frac{1}{x+n}, x \in [0,b], b>0$ be a sequence of real valued function . Then

(a) Pointwise limit of $f_n(x)$ is $f(x)=0$

(b) uniform limit of $f_n(x)$ is $f(x)=0$

(c) It is not uniformly convegent

(d) Both (a) and (b)

B. Fill up the blanks[15(3 from each unit)]

1. No upper sum can ever be _____ any lower sum.

2. A bounded function f defined on $[a,b]$ is R-integrable iff the lower integral _____ the upper integral.

3. Every ____ function is R-integrable.

4. The improper integral $\int_a^{\infty} \frac{dx}{x^n}, (a>0)$ is convergent *iff* _____ .

5. The integral $\int_0^1 x^{m-1}(1-x)^{n-1} dx$ exists *iff* m and n are _____ .

6. The integral $\int_0^\infty x^{n-1}e^{-x}dx$ is convergent *iff* ____ .

7. Uniformly convergent improper integral of a continuous function is ____ a continuous function.

8. The improper integral $\int_{-1}^{1}\frac{\cos yx}{\sqrt{1-x^2}}dx$ ______ uniformly convergent.

9. if $f(x,y)$ and $\frac{\partial}{\partial y}f(x,y)$ are continuous function of x and y for $a \le x \le b, c \le y \le d, a, b$ being independent of y, then $\frac{d}{dx}\int_a^b f(x.y)dx =$________.

10. A simple close curve is called ______ curve.

11. The value of the double integral $\int_1^4\int_0^{\sqrt{y}} e^{\frac{x}{\sqrt{y}}}dxdy$ is _______.

12. The area of the region bounded by the curve $y = x$ and $y = x^2$ is _____.

13. Uniform convergence ______ pointwise convergence.

14. Uniform limit ______ pointwise limit.

15. Let $\langle f_n \rangle$ be a sequence of function on I such that $\lim_{n\to\infty} f_n(x) = f(x), x \in I$ and let $M_n = \sup\{|f_n(x) - f(x)| : x \in I\}$. Then $\langle f_n(x) \rangle$ converges uniformly on I *iff* _______.

Key Answers

A. Multiple choice questions

1.(c)	2.(d)	3.(c)	4.(d)	5.(a)	6.(b)
7. (b)	8.(c)	9.(a)	10.(a)	11.(a)	12.(b)
13.(a)	14.(b)	15.(d)	16.(a)	17.(a)	18.(a)
19.(c)	20.(c)	21.(a)	22.(a)	23.(d)	24.(a)

25.(d)

Fill in the blanks

1.less than

2.Equal to (or =)

3.Continuous

4. $n>1$

5.Both positive ($m>0, n>0$)

6. $n>0$

7.Itself

8.Is

9. $\int_a^b \frac{\partial}{\partial x} f(x,y)dx$

10.Jordan

11. $\frac{14}{3}(e-1)$

12. $\frac{1}{6}$

13.implies

14.equal to

15. $\lim_{n\to\infty} M_n = 0$