2016

(5th Semester)

PHYSICS

FIFTH PAPER

(Mathematical Physics—I)

(Revised)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

1. *(a)* Evaluate the integral

$$\int_0 e^{-ax} x^{m-1} \cos bx \, dx \qquad 4$$

(b) Prove that

$$(m)\ m\ \frac{1}{2} \quad \frac{\sqrt{\ }}{2^{2m-1}}\ (2m) \qquad 6$$

Or

(a) Show that 3+3=6

(i) $\int_0^{/2} \frac{d}{\sqrt{\sin}} \quad \int_0^{/2} \sqrt{\sin}\ d$

(ii) $\int_0 \frac{x^8(1-x^6)}{(1-x)^{24}} dx \quad 0$

(b) Starting with the definition of -function, show that $\frac{1}{2} \quad \sqrt{\ }$. 4

2. *(a)* Show that the function $f(z) \quad u \quad iv$, where

$$f(z) \quad \frac{x^3(1-i) \quad y^3(1-i)}{x^2 \quad y^2};\ (z \quad 0),\ f(0) \quad 0$$

is continuous and that the Cauchy-Riemann equations are satisfied at the origin yet f (0) does not exist. 5

(b) State Taylor's series theorem. Expand

$$f(z) \quad \frac{z \quad 1}{(z \quad 3)(z \quad 4)}$$

about $z \quad 2$. 1+4=5

Or

(a) State and prove Cauchy's residue theorem. 5

(b) Using Cauchy's integral theorem, integrate $\frac{z^2 \quad 1}{z^2 \quad 1}$ along a circle of radius 1 with centre at *(i)* $z \quad 1$ and *(ii)* $z \quad i$. 2½+2½=5

3. (a) Show that the transpose of the product of two matrices is the product of their transposes taken in the reverse order i.e., $(AB)^T \quad B^T A^T$, where T denotes transpose. 3

(b) If H is a Hermitian matrix, show that e^{iH} is a unitary matrix. 3

(c) Show that any two eigenvectors corresponding to two distinct eigen-values of a Hermitian matrix are orthogonal. 4

Or

(a) Find the eigenvalues and eigenvectors of the matrix $\begin{matrix} 1 & 2 \\ 3 & 2 \end{matrix}$. 5

(b) The matrix $A \quad \begin{matrix} a & h \\ h & b \end{matrix}$ is transformed to the diagonal form $D \quad T^{\ 1}AT$, where

$$T \quad \begin{matrix} \cos & \sin \\ \sin & \cos \end{matrix}$$

Find the value of , which gives the diagonal transformation. 5

4. (a) Find the scale factors for cylindrical coordinate systems. Express $\vec{\ } . \vec{A}$ in cylindrical coordinates. 3+2=5

(b) Show that unit vectors in spherical polar coordinate system are related to unit vectors in Cartesian coordinate system as

$$\begin{matrix} \hat{r} & \sin\cos & \sin\sin & \cos & \hat{i} \\ \hat{} & \cos\cos & \cos\sin & \sin & \hat{j} \\ \hat{} & \sin & \cos & 0 & \hat{k} \end{matrix} \quad 5$$

Or

(a) Show that every tensor of rank 2 can be expressed as the sum of symmetric and skew-symmetric tensor. 2

(b) Show that $\frac{A_i}{x_j}$ is not a tensor although A_i is a covariant tensor of rank one. 3

(c) Show that the contraction of the tensor A_q^p is a scalar. 3

(d) Write the transformation relation for the following tensors : 2

(i) A and *(ii)* B

5. *(a)* Explain in briefly about standard input/output stream of C++ with examples. 3

(b) Write an appropriate C++ statement for each of the following : 3

(i) Read the values of *a*, *b* and *c*

(ii) Write the values of *a* and *b* in one line followed by the value of *c* on another line

(iii) Write the values of *a* and *b* in one line separated by blanks and value of *c* after two blank lines

(c) Write a C++ program that reads two values *x* and *y*, exchanges their contents and prints the output. 4

Or

(a) With the help of appropriate flowchart diagram, describe how 'if', 'if else' and 'nested if' control statements are executed in C++ program. 6

(b) Using 'nested if' control statement, write a program that will find and print the greatest of three numbers. 4

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Subject Code : V/ PHY (v) (R)

Booklet No. A

Date Stamp .....

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To be filled in by the Candidate

DEGREE 5th Semester
(Arts / Science / Commerce / .....) Exam., **2016**

Subject .....
Paper .....

INSTRUCTIONS TO CANDIDATES

1. **The Booklet No. of this script should be quoted in the answer script meant for descriptive type questions and vice versa.**
2. **This paper should be ANSWERED FIRST and submitted within 1 (one) Hour of the commencement of the Examination.**
3. **While answering the questions of this booklet, any cutting, erasing, over-writing or furnishing more than one answer is prohibited. Any rough work, if required, should be done only on the main Answer Book. Instructions given in each question should be followed for answering that question only.**

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DEGREE 5th Semester
(Arts / Science / Commerce / .....) Exam., **2016**

Roll No. .....
Regn. No. .....

Subject .....
Paper .....

Descriptive Type
Booklet No. B .....

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2016

(5th Semester)

PHYSICS

FIFTH PAPER

(Mathematical Physics—I)

(Revised)

(PART : A—OBJECTIVE)

(*Marks* : 25)

The figures in the margin indicate full marks for the questions

SECTION—I

(*Marks* : 10)

Put a Tick (✓) mark against the correct answer in the brackets provided : 1×10=10

1. The value of ratio $\frac{\Gamma(1)}{\Gamma(0)}$ is

(a) 1 ( )

(b) ∞ ( )

(c) 0 ( )

(d) $\frac{2}{3}$ ( )

2. The value $\beta(z, 1)$ is

(a) $\frac{1}{z}$ ( )

(b) $\frac{1}{(z+1)}$ ( )

(c) $\frac{1}{z(z+1)}$ ( )

(d) $\frac{z}{(z+1)}$ ( )

3. The equation for a circle with centre at $(-1, 1)$ and radius 3 is

(a) $|z+1+i|=3$ ( )

(b) $|z+1-i|=3$ ( )

(c) $|z-1-i|=3$ ( )

(d) $|z-1+i|=3$ ( )

4. The function $\frac{1}{(z-1)^{1/2}}$

(a) is analytic in the region $|z|<2$ ( )

(b) has a pole at $z=1$ ( )

(c) has a branch point at $z=1$ ( )

(d) has an essential singularity at $z=1$ ( )

5. Rotation of a point (0, 1) using the matrix

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

about the origin through an angle $\pi / 4$ results in a point

(a) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ ( )

(b) $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ ( )

(c) (1, 0) ( )

(d) (0, −1) ( )

6. The rank of $n \times m$ matrix

(a) is the largest square submatrix with non-zero determinant ( )

(b) is the number of columns in $n \times m$ matrix ( )

(c) is the number of rows in $n \times m$ matrix ( )

(d) is the largest square submatrix with zero determinant ( )

7. In cylindrical coordinate system, the intersection between coordinate surfaces $\rho = c_1$ and $z = c_3$ is a/an

(a) straight line ( )

(b) circle ( )

(c) semi-circle ( )

(d) ellipse ( )

8. If x^i, $i = 1, 2$ represents rectangular coordinates, the relation $x^i x^i = 1$ would represent

(a) a circle of unit radius ( )

(b) a sphere of unit radius ( )

(c) a straight line in two dimensions ( )

(d) a hypersphere of unit radius ( )

9. The output of the program

```
for (char i='A'; i<'E'; i++)
        cout <<i;
```

will be

(a) A C ( )

(b) B C D ( )

(c) A B C D ( )

(d) A B C D E ( )

10. In C++ program given below, the value of x after the second statement is

```
int x = 3; // first
x+ = 10; // second
```

(a) 13 ( )

(b) 10 ( )

(c) 16 ( )

(d) 0 ( )

SECTION—II

(*Marks* : 15)

Give short answers to the following questions : 3×5=15

1. Show that $\beta(m, n) = \beta(n, m)$

2. Show that the function $f(z) = u + iv$ with $u = x^2 - y^2$, $v = 2xy$ satisfy Cauchy-Riemann equation.

3. Show that every square matrix can be uniquely expressed as the sum of symmetric and skew-symmetric matrices.

4. Show that the tangent unit vectors are identical to perpendicular unit vectors in orthogonal curvilinear coordinate system.

5. What are structure and string in C++ programming language?

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