2014

(5th Semester)

PHYSICS

SIXTH PAPER

(Quantum Mechanics—II)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

1. *(a)* Obtain de Broglie relation by using Lorentz transformation and the standard wave equation. 7

(b) What do you mean by quantum numbers? Write down the possible quantum numbers for $n = 2$. 3

Or

(a) Show that de Broglie wavelength for a material particle of rest mass m_0 and

charge q, accelerated from rest through a potential difference of V volts relativistically is given by

$$\lambda = \frac{h}{\sqrt{2m_0 qV\left(1+\frac{qV}{2m_0c^2}\right)}}$$ 5

(b) Show that material particle can only be represented by a group wave, not by single wave. 5

2. (a) Show that two eigenfunctions of a Hermitian operator belonging to two distinct eigenvalues are orthogonal. 5

(b) Show that if two Hermitian operators commute, their product is also Hermitian. 5

Or

(a) What do you mean by basis set? Do the following matrices form a basis set for a vector space of 2×2 matrices?

$$M_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix},\ M_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix},\ M_3 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix},\ M_4 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

Justify your answer. 5

(b) Let $|\psi\rangle = 2|u_1\rangle - 3|u_2\rangle + i|u_3\rangle$ and $|\phi\rangle = 3|u_1\rangle - 2|u_2\rangle + 4|u_3\rangle$ and a constant $a = 3+3i$. Compute the inner product $\langle\psi|\phi\rangle$ and $|a\psi\rangle$.

 5

3. *(a)* Obtain the time-independent Schrödinger equation. 6

(b) The wave function of a particle is given by $\psi(x) = Ae^{-a^2x^2}$, $-\infty < x < +\infty$. Obtain the expression for normalization constant and the probability of finding the particle in the region $0 < x < \infty$. 4

Or

(a) Obtain the equation for conservation of probability in quantum mechanics. Write the physical meaning of the equation. 6

(b) What do you mean by expectation values in quantum mechanics? Show that the expectation value of Hamiltonian H is the total energy of the system. 1+3=4

4. What do you mean by Quantum mechanical tunneling effect? Show that the transmittance of a particle incident at rectangular potential barrier is given by

$$T = \frac{16E(V_0 - E)}{V_0^2} e^{-\frac{2\sqrt{2m(V_0 - E)}}{h}a}$$

where V_0 is the potential barrier, a is barrier thickness. 10

Or

A free particle of energy E is incident on a potential step given by $V = 0$; $x < 0$ and $V = V_0$; $x \geq 0$. Obtain the expressions for transmittance and reflectance for the case $E > V_0$.

5. (a) Write down Pauli spin matrices and show that $[\sigma^2, \sigma_x] = 0$. 1+4=5

(b) Show that—

(i) $\sigma_x \sigma_y + \sigma_y \sigma_x = 0$;

(ii) $[\sigma_x, \sigma_y] = 2i\sigma_z$. 2+3=5

Or

(a) Show that commutation relation between X and Y components of angular momentum L_x and L_y is given by $[L_x, L_y] = i\hbar L_z$. 5

(b) What do you mean by orbital gyromagnetic ratio for an electron? Obtain the expression for it. 1+4=5

★ ★ ★

2014

(5th Semester)

PHYSICS

SIXTH PAPER

(Quantum Mechanics—II)

(PART : A—OBJECTIVE)

(*Marks* : 25)

The figures in the margin indicate full marks for the questions

SECTION—I

(*Marks* : 10)

Put a Tick (✓) mark against the correct answer in the brackets provided : 1×10=10

1. Of the following particles moving with the same velocity, the one which has the largest wavelength is

(a) an electron ( )

(b) a proton ( )

(c) a neutron ( )

(d) an α-particle ( )

2. According to Schrödinger, a particle is equivalent to ______.

(a) a single wave ( )

(b) a wave packet ( )

(c) a light wave ( )

(d) Cannot behave as wave ( )

3. Eigenvalues of Hermitian operators

(a) are real only ( )

(b) are imaginary only ( )

(c) can be real or imaginary ( )

(d) are always complex ( )

4. Let $u = (1, 2, 4)$ and $v = (2, -3, 5)$ be any two vectors in R^3 space. Their inner product is equal to

(a) 10 ( )

(b) 7 ( )

(c) –9 ( )

(d) 16 ( )

5. Let ψ be a wave function, the quantity $\psi^* \psi$ represents

(a) probability density ( )

(b) charge density ( )

(c) energy density ( )

(d) wave intensity ( )

6. Conservation of probability in quantum mechanics is represented by the equation

(a) $\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{J} = 0$ ( )

(b) $\frac{\partial \rho}{\partial t} - \vec{\nabla} \cdot \vec{J} = 0$ ( )

(c) $\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{P} = 0$ ( )

(d) $\frac{\partial \rho}{\partial t} - \vec{\nabla} \cdot \vec{P} = 0$ ( )

7. Let E_3 be energy of the third energy level of a free particle in one-dimensional infinite potential well. The relation between first energy level E_1 and third energy level E_3 is

(a) $E_3 = 3E_1$ ( )

(b) $E_3 = E_1$ ( )

(c) $E_1 = 9E_3$ ( )

(d) $E_3 = 9E_1$ ( )

8. For a free particle in step potential, let R and T be reflectance and transmittance, then

(a) $R + T = 1$ ( )

(b) $R = T$ ( )

(c) $R - T = 1$ ( )

(d) $R \cdot T = 1$ ( )

9. The orbital magnetic moment of an electron is given (where L is angular momentum and m is mass of electron) by

(a) $\mu_L = \frac{eL}{2m}$ ( )

(b) $\mu_L = \frac{ne\hbar}{2m}$ ( )

(c) $\mu_L = \frac{neL}{2m}$ ( )

(d) Both *(a)* and *(b)* ( )

10. For electron, the number of possible spin states for Z component is

(a) 1 ( )

(b) 2 ( )

(c) 3 ( )

(d) 4 ( )

SECTION—II

(*Marks* : 15)

Give short answers to the following questions : 3×5=15

1. Show that a material particle cannot be equivalent to a single wave starting from de Broglie relation.

2. Show that $[x, p_x^n] = ni\hbar p_x^{n-1}$, where x is position operator, p_x is x component of momentum operator.

3. What do you mean by eigenvalue and eigenfunction? Is the function e^{ikx} an eigenfunction with respect to the operator $\frac{d}{dx}$? If so, what are the eigenvalue and eigenfunction?

4. The solution for a free particle in an infinitely high potential is given by $\psi = A \sin kx + B \cos kx$. If both the constants A and B are zero each, what physical meaning does it imply?

5. Show that electron spin magnetic moment is equal to Bohr magnetron.

* * *