

2017

(5th Semester)

PHYSICS

SIXTH PAPER

(Quantum Mechanics—II)

(Pre-Revised)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(Marks : 50)

The figures in the margin indicate full marks
for the questions

1. (a) What do you mean by duality of radiation and matter? Show that the de Broglie wavelength for a material particle of rest mass m_0 and charge q , accelerated from rest through a potential difference of V volt relativistically is given by

$$\lambda = \frac{h}{\sqrt{2m_0q^V \left(1 + \frac{q^V}{2m_0c^2}\right)}}$$

2+5=7

- (b) What do you mean by quantum numbers? Write down the possible quantum numbers for $n = 2$. 3

Or

- (a) Write the main features of Bohr's theory of hydrogen atom. Derive the expressions for total energy of the electron. 2+3=5
- (b) Show that material particle can only be represented by a group wave, not by a single wave. 5

2. (a) What do you mean by Hermitian operator? Show that two eigenfunctions of the same Hermitian operator belonging to two distinct eigenvalues are orthogonal. 1+4=5
- (b) What is expectation value? Show that momentum operator is Hermitian. 1+4=5

Or

- (a) Write down the addition and multiplication conditions to be satisfied by a vector space. 4
- (b) Let $|u_1\rangle = \frac{2}{3}|u_1\rangle + \frac{3}{4}|u_2\rangle + i|u_3\rangle$ and $|u_2\rangle = \frac{3}{4}|u_1\rangle + \frac{2}{3}|u_2\rangle + 4|u_3\rangle$ and a constant a such that $\langle u_1 | u_2 \rangle = a$. Compute the inner product $\langle u_1 | a u_2 \rangle$. 3+3=6

(3)

3. (a) Obtain Schrödinger time independent equation. Give the physical interpretation. 6
- (b) A wave function is given by Ae^{ikx} , show that the probability current density of the given wave function is given by $J = v|A|^2$ where v is velocity of the particle. 4
- Or
- (a) Obtain the equation for conservation of probability. Write the physical meaning of the equation. 7
- (b) Write down the conditions to be satisfied by an acceptable wave function. 3

4. Obtain the expression for energy eigenvalue of one-dimensional harmonic oscillator. What do you mean by zero point energy? 8+2=10

Or

A free particle of energy E is incident on a potential step given by $V = 0$; $x < 0$ and $V = V_0$; $x > 0$. Show that all the waves are reflected when $E < V_0$. 10

(4)

5. (a) Write down Pauli spin matrices and show that $[L^2, L_x] = 0$. 1+4=5
- (b) What do you mean by orbital gyromagnetic ratio for an electron? Obtain the expression for it. 1+4=5
- Or
- (a) Show that the square of angular momentum commutes with any one of the components of angular momentum i.e., $[L^2, L_x] = 0$. What is the physical meaning of the commutation? 4+1=5
- (b) Let $\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z$ be Pauli spin matrices. Let $\vec{A}$ and $\vec{B}$ be two vectors. Show that
- $$(\vec{A} \cdot \vec{\sigma})(\vec{B} \cdot \vec{\sigma}) = \vec{A} \cdot \vec{B} + i(\vec{A} \times \vec{B}) \cdot \vec{\sigma} \quad 5$$

Subject Code : PHY/V/06 (PR)

Booklet No. A

Date Stamp

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To be filled in by the Candidate

DEGREE 5th Semester
(Arts / Science / Commerce /
.....) Exam., **2017**

Subject

Paper

INSTRUCTIONS TO CANDIDATES

1. The Booklet No. of this script should be quoted in the answer script meant for descriptive type questions and vice versa.
2. This paper should be **ANSWERED FIRST** and submitted within 1 (one) Hour of the commencement of the Examination.
3. While answering the questions of this booklet, any cutting, erasing, over-writing or furnishing more than one answer is prohibited. Any rough work, if required, should be done only on the main Answer Book. Instructions given in each question should be followed for answering that question only.

To be filled in by the Candidate

DEGREE 5th Semester
(Arts / Science / Commerce /
.....) Exam., **2017**

Roll No.

Regn. No.

Subject

Paper

Descriptive Type

Booklet No. B

*Signature of
Scrutiniser(s)*

*Signature of
Examiner(s)*

*Signature of
Invigilator(s)*

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PHY/V/06 (PR)

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(5th Semester)

PHYSICS

SIXTH PAPER

(Quantum Mechanics—II)

(Pre-Revised)

(PART : A—OBJECTIVE)

(Marks : 25)

The figures in the margin indicate full marks for the questions

SECTION—I

(Marks : 10)

Put a Tick (✓) mark against the correct answer in the
brackets provided : 1×10=10

- 1.** The de Broglie wavelength of electron accelerated through potential difference of 144 V is about

- (a) 12 angstrom ()
(b) 1.2 angstrom ()
(c) 0.10 angstrom ()
(d) 10 angstrom ()

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(2)

2. Which of the following is not permissible set of quantum numbers for electrons in an atom?

(a) $n = 4, l = 0, m = 0, s = -1/2$ ()

(b) $n = 5, l = 3, m = 0, s = +1/2$ ()

(c) $n = 3, l = 2, m = -3, s = -1/2$ ()

(d) $n = 3, l = 2, m = -2, s = -1/2$ ()

3. The eigenvalues of Hermitian operators are

(a) real only ()

(b) imaginary only ()

(c) can be real or imaginary ()

(d) always complex ()

4. If the inner product between two vectors is zero, then the two vectors are

(a) orthogonal to each other ()

(b) parallel to each other ()

(c) Both (a) and (b) ()

(d) Neither (a) nor (b) ()

5. Let ψ be a wave function, the quantity $\int \psi^* \psi d\tau$ represents

- (a) probability density ()
- (b) total probability ()
- (c) energy density ()
- (d) wave intensity ()

6. Conservation of probability in quantum mechanics is represented by the equation

- (a) $\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{J} = 0$ ()
- (b) $\frac{\partial \rho}{\partial t} - \vec{\nabla} \cdot \vec{J} = 0$ ()
- (c) $\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{P} = 0$ ()
- (d) $\frac{\partial \rho}{\partial t} - \vec{\nabla} \cdot \vec{P} = 0$ ()

Where the symbols have their usual meaning.

7. For a free particle in one-dimensional infinite potential, the relation between energy eigenvalue (E_n) and the quantum state (n) is given by

- (a) $E_n \propto n$ ()
- (b) $E_n \propto n^2$ ()
- (c) $E_n \propto \sqrt{n}$ ()
- (d) $E_n \propto \frac{1}{n^2}$ ()

(4)

8. For a free particle in step potential, let R and T be reflectance and transmittance, then

(a) $R + T = 1$ ()

(b) $R = T$ ()

(c) $R - T = 1$ ()

(d) $RT = 1$ ()

9. For electron, the number of possible spin states for Z component is

(a) 1 ()

(b) 2 ()

(c) 3 ()

(d) 4 ()

10. Trace of Pauli spin matrices are

(a) 1 each ()

(b) $-i$ each ()

(c) -1 each ()

(d) 0 each ()

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(5)

SECTION—II

(Marks : 15)

Give short answers to the following questions : 3×5=15

1. Show that group velocity v_g and phase velocity v_p are related to each other by

$$v_g = v_p - \lambda \frac{dv_p}{d\lambda}$$

(6)

2. Show that $[x, p_x^n] = ni\hbar p_x^{n-1}$, where x is position operator, p_x is x component of momentum operator.

(7)

3. Give the physical interpretation of wave function.
What does normalization condition mean?

(8)

4. Normalised wave function of a free particle in a box is given by

$$\psi = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$$

where $0 < x < L$. Obtain the probability of finding the particle within

$$0 < x < \frac{2}{L}$$

(9)

5. Show that electron spin magnetic moment is equal to Bohr Magneton.

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