Professional Course (Even) Examination, 2025

(2nd Semester)

BACHELOR OF COMPUTER APPLICATIONS

(Discrete Mathematics)

Full Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

(PART : A—OBJECTIVE)

(*Marks* : 25)

SECTION—I

(*Marks* : 15)

I. Tick (✓) the correct answer in the brackets provided : 1×10=10

1. Two finite sets A and B are said to be equivalent if

   (a) $A \cap B = \phi$ ( )

   (b) $n(A) = n(B)$ ( )

   (c) $A = B$ ( )

   (d) $A \cap B \neq \phi$ ( )

2. In Boolean algebra, $a + (a \cdot b)$ is equal to

   (a) a ( )

   (b) b ( )

   (c) $a + b$ ( )

   (d) $a - b$ ( )

3. A statement formula which is neither a tautology nor a contradiction is called

(a) logical equivalence ( )

(b) logical consequence ( )

(c) contingent ( )

(d) conjunction ( )

4. Which of the following is not a statement?

(a) 4 is a real number ( )

(b) I am a liar ( )

(c) Delhi is the capital of Japan ( )

(d) Barack Obama is the President of USA ( )

5. The value of ${}^{15}C_3$ is

(a) 540 ( )

(b) 650 ( )

(c) 255 ( )

(d) 455 ( )

6. The number of different 4-letter words (may be meaningless) that can be formed from the letters of the word 'NUMBERS' is

(a) 840 ( )

(b) 720 ( )

(c) 5040 ( )

(d) 650 ( )

7. If $\gcd(3, 4) = 1$, then 3 and 4 are

(a) relatively prime integers ( )

(b) commutative ( )

(c) associative ( )

(d) None of the above ( )

8. $\phi(12) = ?$

(a) 4 ( )

(b) 12 ( )

(c) 3 ( )

(d) 2 ( )

9. A graph in which each edge is assigned a numerical value is called

(a) subgraph ( )

(b) weighted graph ( )

(c) Hamiltonian graph ( )

(d) simple graph ( )

10. A tree with n vertices has

(a) n edges ( )

(b) no edge ( )

(c) 1 edge ( )

(d) $(n-1)$ edges ( )

II. State whether the following are *True (T)* or *False (F)* : 1×5=5

1. The set of all points on a line segment is a finite set. ( )

2. The operational symbols $\sim, \wedge, \vee, \models, \equiv, \rightarrow$ and $\leftrightarrow$ are called connectives. ( )

3. The number of all permutations of n different things taken all at a time given by ${}^nP_n = n!$. ( )

4. If $a > 0$, then $\gcd(a, a) = 0$. ( )

5. A graph which is permitted to have multiple edges is called multigraph. ( )

SECTION—II

(*Marks* : 10)

III. Answer the following questions : 2×5=10

1. *(a)* Prove that $a + a = a$ and $a \cdot a = a$ for every $a \in B$.

OR

(b) If $A = \{a, b, c, d, e\}$, $B = \{a, c, e, g\}$ and $C = \{b, e, f, g\}$, then verify that $A \cap (B - C) = (A \cap B) - (A \cap C)$.

2. *(a)* Prove that $p \to q, \sim p \to q \models q$.

OR

(b) Verify that $(p \wedge q) \wedge \sim (p \vee q)$ is a contradiction.

3. *(a)* Expand $(x^2 + 2y)^5$ by binomial theorem.

OR

(b) If ${}^{18}C_r = {}^{18}C_{r+2}$, then find the value of ${}^rC_5$.

4. *(a)* Solve $5x \equiv 2 \pmod 9$.

OR

(b) Prove that $\gcd(a, b) = \gcd(|a|, |b|)$.

5. *(a)* Define Eulerian graph and tree.

OR

(b) Define simple graph with example.

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

IV. Answer the following questions : $10 \times 5 = 50$

1. *(a)* In a class, 18 students offered physics, 23 offered chemistry and 24 offered mathematics. Of these, 13 are in both chemistry and mathematics, 12 in physics and chemistry, 11 in mathematics and physics, and 6 in all the three subjects. Find—

(i) how many students are there in the class;

(ii) how many offered mathematics but not chemistry;

(iii) how many are taking exactly one of the three subjects. 6

(b) If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6, 8\}$ and $C = \{1, 4, 5, 6\}$, then find—

(i) $(B')'$;

(ii) $(A \cup B)'$;

(iii) $(A \cap C)'$;

(iv) $(B - C)'$. 4

OR

(c) Draw switching circuit for the following Boolean expression : 4

$$z(x + y') + xz' + z'(z + y')$$

(d) Construct switching table for the switching function f represented by the Boolean expression $xyz + x'(y + z)$. 6

2. *(a)* Construct a truth table for the statement formula

$$(p \wedge \sim q) \vee (q \wedge (\sim p \vee r))$$ 5

(b) Verify by truth tables $p \rightarrow (q \wedge r) \equiv (p \rightarrow q) \wedge (p \rightarrow r)$. 5

OR

(c) Determine whether the following statement is tautology, contradiction or contingent : 5

$$\sim((p \rightarrow q) \leftrightarrow \sim(p \wedge \sim q))$$

(d) Construct truth table for the following : 5

$$(p \rightarrow (\sim q \vee r)) \wedge (q \vee (p \leftrightarrow r))$$

3. *(a)* Find the 4th term from the end in the expansion of

$$\left(\frac{4x}{5} - \frac{5}{2x}\right)^9$$ 5

(b) Find the term independent of x in the expansion of

$$\left(x^2 - \frac{2}{x^3}\right)^{15}$$ 5

OR

(c) Find the coefficient of x^6 in the expansion of

$$\left(3x^2 - \frac{1}{3x}\right)^9$$ 5

(d) Find the two middle terms in the expansion of

$$\left(x^4 - \frac{1}{x^3}\right)^{11}$$ 5

4. *(a)* State and prove Fermat's theorem. 5

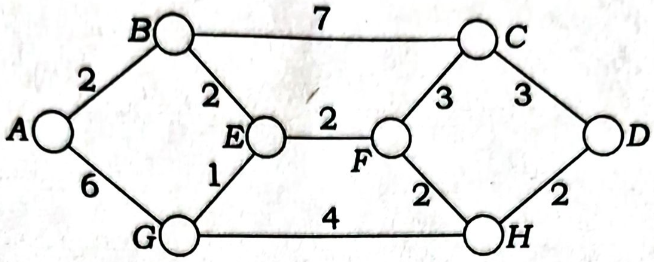
(b) Prove that the relation 'congruence modulo m' is an equivalence relation in the set of integers. 5

OR

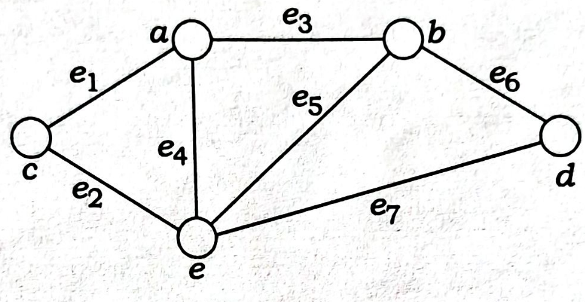
(c) Find (275, 200) and express it in the form of $275x + 200y$, where $x, y \in Z$. 5

(d) State and prove Euler's theorem. 5

5. *(a)* Using Kruskal's algorithm, find the minimum spanning tree for the weighted graph of the following figure : 6

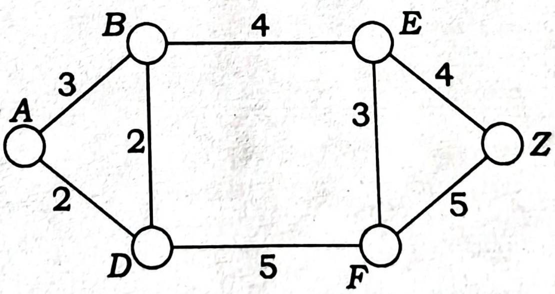


(b) Write the incidence matrix of the following graph : 4

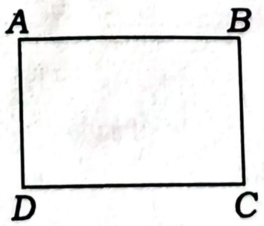


OR

(c) Using Prim's algorithm, find the minimum spanning tree for the weighted graph of the following figure : 6



(d) Find the chromatic polynomial and chromatic number for the graph of the following figure : 4



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