MATH100 (MAJOR) **Student's Copy**

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(NEP–2020)

(1st Semester)

MATHEMATICS (MAJOR)

(Vector Analysis)

Full Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

(SECTION : A—OBJECTIVE)

(*Marks* : 10)

Tick (✓) the correct answer in the brackets provided : 1×10=10

1. For any two vectors $\vec{u}$ and $\vec{v}$, if $\vec{v} \quad \frac{d\vec{u}}{dt}$, then $\frac{d}{dt} \quad \vec{u} \quad \frac{d\vec{u}}{dt}$ is equal to

(a) 0 ( )

(b) $\frac{d\vec{u}}{dt} \quad \frac{d\vec{u}}{dt}$ ( )

(c) $\vec{u} \quad \frac{d\vec{u}}{dt}$ ( )

(d) $\vec{u} \quad \frac{d^2\vec{u}}{dt^2}$ ( )

2. The derivative of any constant vector *c* is

(a) 2 ( )

(b) *c* ( )

(c) 0 ( )

(d) 1 ( )

3. If $\vec{r} \quad \sin t\,\hat{i} \quad \cos t\,\hat{j} \quad t\,\hat{k}$, then the value of the acceleration is

(a) $\cos t\,\hat{i} \quad \sin t\,\hat{j} \quad \hat{k}$ ( )

(b) $\sin t\,\hat{i} \quad \cos t\,\hat{j} \quad \hat{k}$ ( )

(c) $\sin t\,\hat{i} \quad \cos t\,\hat{j}$ ( )

(d) $\sin t\,\hat{i} \quad \cos t\,\hat{j}$ ( )

4. If $\vec{a}$ is a constant vector, then grad $(\vec{a} \quad \vec{r})$ is equal to

(a) $\vec{a} \quad \vec{r}$ ( )

(b) $\vec{r}$ ( )

(c) $\vec{a}$ ( )

(d) 0 ( )

5. If $(x, y, z) \quad x^2 \quad y^2 \quad z^2$, then the value of grad at the point (1, 2, 3) is

(a) $2\hat{i} \quad 3\hat{j} \quad 6\hat{k}$ ( )

(b) $2\hat{i} \quad 4\hat{j} \quad 6\hat{k}$ ( )

(c) $3\hat{i} \quad 4\hat{j} \quad 5\hat{k}$ ( )

(d) $3\hat{i} \quad 2\hat{j} \quad 5\hat{k}$ ( )

6. If $\vec{F} \quad (5xy \quad 6x^2)\hat{i} \quad (2y \quad 4x)\hat{j}$, then the value of $_C \vec{F} \, d\vec{r}$ along the curve C in the xy-plane, $y \quad x^3$ from the point (1, 1) to (2, 8), is

(a) 25 ( )

(b) 15 ( )

(c) 45 ( )

(d) 35 ( )

7. The circulation of $\vec{F}$ around the curve C, where $\vec{F} \quad y\hat{i} \quad z\hat{j} \quad x\hat{k}$ and C is the circle $x^2 \quad y^2 \quad 1, z \quad 0$, is

(a) ( )

(b) ( )

(c) 2 ( )

(d) 2 ( )

8. If V is the closed region bounded by the planes $x \quad 0$, $y \quad 0$, $z \quad 0$ and $2x \quad 2y \quad z \quad 4$, then the value of $\underset{V}{\quad} \vec{F}\, d\vec{V}$ is

(a) $\frac{8}{3}$ ( )

(b) $\frac{3}{8}$ ( )

(c) $\quad \frac{8}{3}$ ( )

(d) $\frac{5}{3}$ ( )

9. If $\vec{F}$ is a continuously differentiable vector point function in a region V and S is a closed surface enclosing V, then $\vec{F} \quad \hat{n}\, d\vec{S}$ is equal to

(a) div $\vec{F}\, d\vec{V}$ ( )

(b) div $\vec{F}\, d\vec{V}$ ( )

(c) curl $\vec{F}\, d\vec{V}$ ( )

(d) curl $\vec{F}\, d\vec{V}$ ( )

10. If C is a simple closed curve in the xy-plane not enclosing the origin, then the value of $\underset{C}{\quad} \vec{F}\, d\vec{r}$, where $\vec{F} \quad \frac{y\hat{i} \quad x\hat{j}}{x^2 \quad y^2}$, is

(a) $x^2 \quad y^2$ ( )

(b) $\quad y\hat{i} \quad x\hat{j}$ ( )

(c) 0 ( )

(d) $\frac{y\hat{i} \quad x\hat{j}}{x^2 \quad y^2}$ ( )

(SECTION : B—SHORT ANSWERS)

(*Marks* : 15)

Answer *five* questions, taking at least *one* from each Unit : 3×5=15

UNIT—I

1. Prove that a necessary and sufficient condition that a proper vector $\vec{u}$ has a constant length is $\vec{u} \quad \frac{d\vec{u}}{dt} \quad 0$.

2. A particle moves along the curve $x \quad 2t^2$, $y \quad t^2 \quad 4t$, $z \quad t \quad 5$, where t is the time. Find the components of its velocity and acceleration at time $t \quad 1$ in the direction of $\hat{i} \quad 2\hat{j} \quad 2\hat{k}$.

UNIT—II

3. Show that $\;^2 \; \frac{1}{r} \quad 0$.

4. Find the directional derivative of $(x, y, z) \quad 4xz^3 \quad 3x^2y^2z$ at $(2, \quad 1, 2)$ in the direction of $2\hat{i} \quad 3\hat{j} \quad 6\hat{k}$.

UNIT—III

5. If $\vec{A} \quad (2y \quad 3)\hat{i} \quad xy\hat{j} \quad (yz \quad x)\hat{k}$, evaluate $\;_C \; \vec{A} d\vec{r}$, along the paths C, $x \quad 2t^2$, $y \quad t$, $z \quad t^3$ from $t \quad 0$ to $t \quad 1$.

6. Evaluate $\;_S \; \vec{A} \; \vec{n} \; d\vec{S}$, where $\vec{A} \quad z\hat{i} \quad x\hat{j} \quad 3y^2z\hat{k}$ and S is the surface of the cylinder $x^2 \quad y^2 \quad 16$ included in the first octant between $z \quad 0$ and $z \quad 5$.

UNIT—IV

7. Evaluate by Stokes' theorem

$$\underset{C}{} (e^x dx \quad 2y\, dy \quad dz)$$

where C is the curve $x^2 \quad y^2 \quad 4,\ z \quad 2$.

8. If S is any closed surface enclosing a volume V and $\vec{F} \quad x\hat{i} \quad 2y\hat{j} \quad 3z\hat{k}$, show that $\underset{S}{} \vec{F} \quad \vec{n}\, d\vec{S} \quad 6V$.

(SECTION : C—DESCRIPTIVE)

(*Marks* : 50)

Answer *five* questions, taking at least *one* from each Unit : 10×5=50

UNIT—I

1. *(a)* If $\vec{w}$ is a constant vector, $\vec{r}$ and $\vec{s}$ are vector functions of a scalar variable t and if $\frac{d\vec{r}}{dt} \quad \vec{w} \quad \vec{r},\ \frac{d\vec{s}}{dt} \quad \vec{w} \quad \vec{s}$, then show that

$$\frac{d}{dt}(\vec{r} \quad \vec{s}) \quad \vec{w} \quad (\vec{r} \quad \vec{s})$$ 4

(b) Find $\vec{T}, \vec{N}, k$ for the plane curve $\vec{r}(t) \quad t\hat{i} \quad (\log\cos t)\hat{j},\ \frac{}{2} \quad t \quad \frac{}{2}$. 3

(c) Find the unit tangent vector and arc length of the curve

$$\vec{r}(t) \quad 2\cos t\hat{i} \quad 2\sin t\hat{j} \quad \sqrt{5}\, t\hat{k},\ 0 \quad t$$ 3

2. *(a)* If $\vec{r}(t) \quad a\cos t\hat{i} \quad a\sin t\hat{j} \quad at\tan \quad \hat{k}$, then find the values of

$$\left|\frac{d\vec{r}}{dt} \quad \frac{d^2\vec{r}}{dt^2}\right| \text{ and } \frac{d\vec{r}}{dt} \quad \frac{d^2\vec{r}}{dt^2} \quad \frac{d^3\vec{r}}{dt^3}$$ 5

(b) Suppose $(x, y, z) \quad xy^2z$ and $\vec{A} \quad xz\hat{i} \quad xy^2\hat{j} \quad yz^2\hat{k}$. Find $\frac{3}{x^2 \quad z}(\quad \vec{A})$ at the point $(2, \quad 1, 1)$. 5

UNIT—II

3. *(a)* Show that $^{2}r^{n} \quad n(n \quad 1)r^{n \quad 2}$, where n is constant. 6

(b) Let $\vec{A} \quad (6xy \quad z^3, 3x^2 \quad z, 3xz^2 \quad y)$. Show that $\vec{A}$ is irrotational and that $\vec{A} \quad$ grad $\quad$, for some scalar point function $\quad$. Find $\quad$. 4

4. *(a)* Find $(\vec{A} \quad) \vec{B}$ at the point $(1, \quad 1, 2)$, if $\vec{A} \quad xz^2\hat{i} \quad 2y\hat{j} \quad 3xz\hat{k}$ and $\vec{B} \quad 3xz\hat{i} \quad 2yz\hat{j} \quad z^2\hat{k}$. 4

(b) If $\vec{w}$ is a constant vector and $\vec{v} \quad \vec{w} \quad \vec{r}$, then prove that $\vec{w} \quad \frac{1}{2}\text{curl}\,\vec{v}$. 3

(c) Prove that $\text{div}(r^n\vec{r}) \quad (n \quad 3)r^n$. 3

UNIT—III

5. *(a)* Evaluate $\quad_{C} \vec{F} \quad d\vec{r}$, where $\vec{F} \quad xy\hat{i} \quad (x^2 \quad y^2)\hat{j}$ and C is the x-axis from $x \quad 2$ to $x \quad 4$ and the line $x \quad 4$ from $y \quad 0$ to $y \quad 12$. 5

(b) Evaluate $\quad_{V} \vec{F} \, dV$, where $\vec{F} \quad 2xz\hat{i} \quad x\hat{j} \quad y^2\hat{k}$ and V is the region bounded by the surfaces $x \quad 0$, $y \quad 0$, $z \quad x^2$ and $z \quad 4$. 5

6. *(a)* Evaluate $\quad_{C} \{yz\,dx \quad (xz \quad 1)\,dy \quad xy\,dz\}$, where C is any path from $(1, 0, 0)$ to $(2, 1, 4)$. 5

(b) Evaluate $\quad_{S} \vec{F} \quad \vec{n} \quad dS$, where $\vec{F} \quad yz\hat{i} \quad zx\hat{j} \quad xy\hat{k}$ and S is that part of the surface of the sphere $x^2 \quad y^2 \quad z^2 \quad 1$ which lies in the first octant. 5

UNIT—IV

7. *(a)* Verify Stokes' theorem for $\vec{A} \quad (y \quad z \quad 2)\hat{i} \quad (yz \quad 4)\hat{j} \quad xz\hat{k}$, where S is the surface of the cube $x \quad 0$, $y \quad 0$, $z \quad 0$ and $x \quad 2$, $y \quad 2$, $z \quad 2$ above the xy-plane. 5

(b) Show that

$$\underset{S}{} (ax\hat{i} \quad by\hat{j} \quad cz\hat{k}) \; \vec{n} \; dS \quad \frac{4}{3} \quad (a \quad b \quad c)$$

where S is the surface of the sphere $x^2 \quad y^2 \quad z^2 \quad 1$. 5

8. *(a)* State Stokes' theorem and hence apply for $\vec{F} \quad y\hat{i} \quad z\hat{j} \quad x\hat{k}$, where S is the upper half surface of the sphere $x^2 \quad y^2 \quad z^2 \quad 1$ and C is its boundary. 5

(b) Evaluate by Green's theorem

$$\underset{C}{} (x^2 \quad \cosh y)\, dx \quad (y \quad \sin x)\, dy$$

where C is the rectangle with vertices (0, 0), (, 0), (, 1), (0, 1). 5

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