**2 0 2 4**

( NEP–2020 )

( 1st Semester )

## MATHEMATICS (MAJOR/MINOR)

### ( Calculus )

*Full Marks* : 75

*Time* : 3 hours

*The figures in the margin indicate full marks for the questions*

### ( SECTION : A—OBJECTIVE )

( *Marks* : 10 )

Put a Tick (✓) mark against the correct answer in the brackets provided : 1×10=10

**1.** The graph of the function $x(y \quad 2) \quad 1$

*(a)* crosses the line $y \quad 2$ ( )

*(b)* crosses the $Y$-axis ( )

*(c)* crosses the $X$-axis at $\quad 1/2$ ( )

*(d)* crosses the line $y \quad 2 \quad x$ ( )

**2.** The gradient of the curve $y \quad 3x^2 \quad 2x \quad 5$ at $x \quad 0$ is

(a) 0 ( )

(b) 1 ( )

(c) 1 ( )

(d) 2 ( )

**3.** If $f(x) \quad [x]$ (the greatest integer function), then

(a) $\lim_{x \quad 1} f(x)$ exists ( )

(b) $f(x)$ is continuous at $x \quad 2$ ( )

(c) $f(x)$ is continuous at $x \quad 2\ 5$ ( )

(d) $f(x)$ is derivable at $x \quad 0$ ( )

**4.** Geometrical interpretation of Rolle's theorem says that there exists a point such that the slope of the tangent is 0 at that point which means the tangent to the curve at that point is

(a) parallel to $x$-axis ( )

(b) perpendicular to $x$-axis ( )

(c) Neither parallel nor perpendicular to $x$-axis ( )

(d) None of the above ( )

**5.** If $a$ and $b$ are two roots of the equation $f(x) \quad 0$, then $f \ (x) \quad 0$ will have at least one root between $a$ and $b$, provided

*(a)* $f(x)$ is derivable in $[a, b]$ ( )

*(b)* $f(x)$ is continuous in $[a, b]$ ( )

*(c)* $f \ (x)$ does not exist in $(a, b)$ ( )

*(d)* None of the above ( )

**6.** If $f(x) \quad {}_{x}^{x^2} \, e^t dt$, then $\dfrac{df(x)}{dx}$ is equal to

*(a)* $2x^2 e^{x^2} \quad e^{x^2}$ ( )

*(b)* $2xe^{x^2} \quad e^x$ ( )

*(c)* $2x^2 e^{x^2} \quad e^{x^2}$ ( )

*(d)* None of the above ( )

**7.** If $f(2a \quad x) \quad f(x)$, then the value of ${}_{0}^{2a} f(x)\,dx$ is equal to

*(a)* $2 \, {}_{0}^{a} f(x)\,dx$ ( )

*(b)* ${}_{0}^{a} f(x)\,dx$ ( )

*(c)* 2 ( )

*(d)* 0 ( )

**8.** If $u \quad f(y \quad z, z \quad x, x \quad y)$, then the value of $\frac{u}{x} \quad \frac{u}{y} \quad \frac{u}{z}$ is

*(a)* 0 ( )

*(b)* 3 ( )

*(c)* $\frac{f}{x} \quad \frac{f}{y} \quad \frac{f}{z}$ ( )

*(d)* None of the above ( )

**9.** Let $f(x, y) \quad \frac{x \quad y}{x^2 \quad y^2}$. Then $\lim f(x, y)$ as $(x, y) \quad (1, 2)$ along the line $y \quad 2$ is

*(a)* $\frac{1}{5}$ ( )

*(b)* $\frac{3}{5}$ ( )

*(c)* $\frac{1}{5}$ ( )

*(d)* Does not exist ( )

**10.** The sequence $_{n \quad 1} \frac{1}{n^p}$ is convergent if

*(a)* $p \quad 1$ ( )

*(b)* $p \quad 1$ ( )

*(c)* $p \quad 1$ ( )

*(d)* For any value of $p$ ( )

## ( SECTION : B—SHORT ANSWERS )

( *Marks* : 15 )

Answer *five* questions, taking at least *one* from each Unit :

3×5=15

### UNIT—I

**1.** Evaluate the limit $\lim_{x} (x \quad e^{x})^{2/x}$.

**2.** Show that a function which is derivable at a point is necessarily continuous at that point.

### UNIT—II

**3.** Expand $e^{x}$ in the power of $(x \quad 3)$.

**4.** Verify Rolle's theorem for the function $f(x) \quad x\sqrt{a^2 \quad x^2}$ in $[0, a]$.

### UNIT—III

**5.** Evaluate the integral $e^{x} \dfrac{x \quad 1}{(x \quad 1)^3} dx$.

**6.** Using the reduction formula for $_0^{/2} \cos^n x\, dx$, evaluate $_0^{/2} \cos^4 x \sin^2 x\, dx$.

### UNIT—IV

**7.** Show that the sequence $\dfrac{(3n)!}{(n!)^3}^{1/n}$ converges and find its limit.

**8.** Let $f : \mathbb{R}^2 \quad \mathbb{R}$ be defined by

$$f(x, y) \quad \begin{matrix} \dfrac{xy}{x^2 \quad y^2}, & (x, y) \quad (0, 0) \\ 0 \quad , & (x, y) \quad (0, 0) \end{matrix}$$

Test the continuity of $f$ at the origin.

## ( SECTION : C—DESCRIPTIVE )

( *Marks* : 50 )

Answer *five* questions, taking at least *one* from each Unit : 10×5=50

### UNIT—I

**1.** *(a)* Draw the graph of the function $f$ defined by

$$f(x) \quad \begin{cases} \frac{|x| \quad x}{2}, & 2 \quad x \quad 0 \\ x \quad , & 0 \quad x \quad 1 \\ 2x \quad , & 1 \quad x \quad 2 \end{cases}$$

From the graph, can you conclude that $f(x)$ is continuous in the interval [ 2, 2] ? 5+1=6

*(b)* The relation between volume ($V$) and presssure ($P$) of a gas is given by $V \quad \frac{200}{P}$. Find the average rate of change of volume with respect to pressure when $P$ increases from 30 to 35. Also find the instantaneous rate of change of volume at $P \quad 30$. 4

**2.** *(a)* Use - definition of continuity to prove that $y \quad \sin x$ is continuous at every value of $x$. 4

*(b)* If $y \quad (x \quad \sqrt{1 \quad x^2})^m$, then prove that

$$(1 \quad x^2)y_{n \ 2} \quad (2n \quad 1)xy_{n \ 1} \quad (n^2 \quad m^2)y_n \quad 0$$

Also find $y_n(0)$. 4+2=6

### UNIT—II

**3.** *(a)* State and prove Cauchy's mean value theorem. 5

*(b)* Expand $\cos x$ in a finite series in power of $x$, with remainder in Cauchy's form. 5

4. *(a)* State and prove Lagrange's mean value theorem. 5

*(b)* Expand $(1+x)^m$ in a finite series with remainder in Cauchy's form. 5

## UNIT—III

5. *(a)* Evaluate $\int_1^2 \frac{1}{x^2}\,dx$ using definite integral as limit of a sum. 5

*(b)* Obtain a reduction formula for $\int \sec^n x\,dx$ and hence find the value of

$$\int \sec^6 x\,dx$$ 5

6. *(a)* Obtain the reduction formula

$$\int_0^{\pi/2} \sin^n x\,dx = \begin{cases} \dfrac{(n-1)(n-3)(n-5)\ldots 3\cdot 1}{n(n-2)(n-4)\ldots 4\cdot 2}\cdot\dfrac{\pi}{2}, & n \text{ is even} \\[2ex] \dfrac{(n-1)(n-3)(n-5)\ldots 4\cdot 2}{n(n-2)(n-4)\ldots 3\cdot 1}\cdot 1, & n \text{ is odd} \end{cases}$$ 6

*(b)* Show that $\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}\,dx = \frac{\pi^2}{4}$. 4

## UNIT—IV

7. *(a)* State and prove Cauchy's general principle of convergence for a sequence. 5

*(b)* If $u = \frac{1}{\sqrt{x^2+y^2+z^2}}$, $x^2+y^2+z^2 \neq 0$, then prove that $u(x, y, z)$ is a harmonic function. 5

8. *(a)* State and prove Euler's theorem on a homogeneous function for three variables. 5

*(b)* Test the convergence or divergence of the series

$$\sum_{n=0}^{\infty} \left(\sqrt[3]{n^3+1} - n\right)$$ 5

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