2 0 2 4

(NEP–2020)

(2nd Semester)

MATHEMATICS (MAJOR/MINOR)

(Algebra)

Full Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

(SECTION : A—OBJECTIVE)

(*Marks* : 10)

Tick (✓) the correct answer in the brackets provided : 1×10=10

1. If $f(x)$ and $g(x)$ are polynomials of degrees m and n respectively, then degree of $f(x)$ $g(x)$ is

(a) m n ( )

(b) m / n ( )

(c) m n ( )

(d) m n ( )

2. The value of k for which a polynomial $x^4 \quad x^3 \quad kx \quad 5$ is divisible by $(x \quad 1)$ is

(a) 4 ( )

(b) 5 ( )

(c) 6 ( )

(d) 7 ( )

3. The polynomial $x^2 \quad x \quad 1$ is

(a) reducible over R ( )

(b) irreducible over Q ( )

(c) irreducible over C ( )

(d) None of the above ( )

4. The equation

$$x^5 \quad 2x^4 \quad 2x^3 \quad 4x^2 \quad x \quad 2 \quad 0$$

has

(a) no multiple root ( )

(b) one multiple root ( )

(c) two multiple roots ( )

(d) three multiple roots ( )

5. Every algebraic equation of odd degree has

(a) at least one real root ( )

(b) at least one complex root ( )

(c) at least one surd root ( )

(d) None of the above ( )

6. If the signs of the terms of an equation are all positive, then

(a) it has at least one positive root ( )

(b) it cannot have a positive root ( )

(c) it cannot have a negative root ( )

(d) it cannot have any real root ( )

7. The roots of the equation $x^3 \quad 5x^2 \quad 4x \quad 20 \quad 0$, given that two of its roots are equal in magnitude but are of opposite signs, are

(a) 4, 5 and 5 ( )

(b) 4, 20 and 20 ( )

(c) 5, 4 and 4 ( )

(d) 5, 2 and 2 ( )

8. If , , are the roots of the equation $x^3 \quad px^2 \quad qx \quad r \quad 0$, then $^2 \quad ^2 \quad ^2$ equals

(a) $p^2 \quad 2q$ ( )

(b) $p^2 \quad 2q$ ( )

(c) $p^2 \quad 2q$ ( )

(d) $p^2 \quad 2q$ ( )

9. If $n \quad 5$ and $\frac{}{10}$, then the value of $(\cos \quad i\sin \quad)^n$ is

(a) i ( )

(b) i ( )

(c) 0 ( )

(d) 1 ( )

10. The expansion of $\sin n$ in powers of $\cos$ and $\sin$, n being a positive integer, is

(a) $n\cos^{n\quad 1}\quad \sin \quad \dfrac{n(n\quad 1)(n\quad 2)}{3!}\cos^{n\quad 3}\quad \sin^{3}\quad \dfrac{n(n\quad 1)(n\quad 2)(n\quad 3)(n\quad 4)}{5!}$

$\cos^{n\quad 5}\quad \sin^{5}\quad \cdots$ ( )

(b) $n\cos^{n\quad 1}\quad \sin \quad \dfrac{n(n\quad 1)(n\quad 2)}{3!}\cos^{n\quad 3}\quad \sin^{3}\quad \dfrac{n(n\quad 1)(n\quad 2)(n\quad 3)(n\quad 4)}{5!}$

$\cos^{n\quad 5}\quad \sin^{5}\quad \cdots$ ( )

(c) $n\cos^{n\quad 1}\quad \sin \quad \dfrac{n(n\quad 1)(n\quad 2)}{3!}\cos^{n\quad 3}\quad \sin^{3}\quad \dfrac{n(n\quad 1)(n\quad 2)(n\quad 3)(n\quad 4)}{5!}$

$\cos^{n\quad 5}\quad \sin^{5}\quad \cdots$ ( )

(d) $n\cos^{n\quad 1}\quad \sin \quad \dfrac{n(n\quad 1)(n\quad 2)}{3!}\cos^{n\quad 3}\quad \sin^{3}\quad \dfrac{n(n\quad 1)(n\quad 2)(n\quad 3)(n\quad 4)}{5!}$

$\cos^{n\quad 5}\quad \sin^{5}\quad \cdots$ ( )

(SECTION : B—SHORT ANSWERS)

(*Marks* : 15)

Answer *five* questions, taking at least *one* from each Unit : 3×5=15

UNIT—I

1. If a polynomial $f(x)$ of degree $n\quad 2$ is divided by $(x\quad\quad)^2$, prove that the remainder is $(x\quad\quad)f'(\;)\quad f(\;)$.

2. Show that $g(x)\quad 3x^4\quad 15x^2\quad 10$ is irreducible over the field of rational numbers.

UNIT—II

3. Show that 2 is a multiple root of $x^3 \quad x^2 \quad 16x \quad 20 \quad 0$ of multiplicity 2.

4. Solve the equation $x^4 \quad 10x^3 \quad 29x^2 \quad 22x \quad 4 \quad 0$, given that one root is $(2 \quad \sqrt{3})$.

UNIT—III

5. Prove that the equation $x^{10} \quad 4x^6 \quad x^4 \quad 2x \quad 3 \quad 0$ has at least four imaginary roots.

6. If the sum of two roots of the equation $x^3 \quad a_1x^2 \quad a_2x \quad a_3 \quad 0$ be zero, show that $a_1a_2 \quad a_3 \quad 0$.

UNIT—IV

7. Find the values of the expression $(1 \quad i)^{1/7}$ using De Moivre's theorem.

8. Expand $\cos 5$ in powers of $\cos$.

(SECTION : C—DESCRIPTIVE)

(*Marks* : 50)

Answer *five* questions, taking at least *one* from each Unit : 10×5=50

UNIT—I

1. *(a)* State and prove the division algorithm for polynomials. 1+7=8

(b) Find the quotient polynomial and the remainder when $x^3 \quad 3x^2 \quad 2x \quad 5$ is divided by $(x \quad 2)$. 2

2. *(a)* State and prove the remainder theorem. 1+2=3

(b) If a polynomial $f(x)$ is divided by $(x \quad)(x \quad)$, , prove that the remainder is

$$\frac{(x \quad)f(\ ) \quad (x \quad)f(\ )}{(\quad)}$$

Using this result, find the remainder when $x^5 \quad 3x^4 \quad 4x^2 \quad x \quad 4$ is divided by $(x \quad 1)(x \quad 2)$. 5+2=7

UNIT—II

3. State the fundamental theorem of algebra. Using it, prove that every algebraic equation of n-th degree has n roots, real or imaginary, and no more. 1+9=10

4. *(a)* Show that if an equation $f(x)\ \ 0$, whose coefficients are all rational quantities, has a surd root of the form $(\ \ \sqrt{\ })$, then the conjugate surd $(\ \ \sqrt{\ })$ is also a root of the same equation. 5

(b) Show that if an equation $f(x)\ \ 0$ whose coefficients are all real quantities, has a complex number of the form $(\ \ i\ )$ as one of its roots, then the conjugate complex number $(\ \ i\ )$ is also a root of the same equation. 5

UNIT—III

5. *(a)* Prove that the equation $x^3\ \ x^2\ \ 5x\ \ 1\ \ 0$ has one positive root lying in $(1, 2)$ and two negative roots lying in $(\ 1, 0)$ and $(\ 3,\ \ 2)$. 4

(b) Show that if the equation $x^3\ \ ax^2\ \ bx\ \ c\ \ 0$ has a pair of roots of the form $(1\ \ i)$, where is real and $i\ \ \sqrt{\ 1}$, then $(a^2\ \ 2b)(b^2\ \ 2ac)\ \ c^2$. 6

6. *(a)* If , , , are the roots of the equation $x^4\ \ px^3\ \ qx^2\ \ rx\ \ s\ \ 0$, show that $(1\ \ ^2)(1\ \ ^2)(1\ \ ^2)(1\ \ ^2)\ \ (1\ \ q\ \ s)^2\ \ (p\ \ r)^2$. 5

(b) If , , are the roots of the equation $2x^3\ \ x^2\ \ x\ \ 1\ \ 0$, find the equation whose roots are $\frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}, \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}, \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}$. 5

UNIT—IV

7. State and prove De Moivre's theorem. 1+9=10

8. *(a)* Solve $x^3\ \ 30x\ \ 133\ \ 0$ by Cardan's method. 5

(b) Solve the cubic equation $x^3\ \ 6x\ \ 9\ \ 0$ by Cardan's method. 5

★★★