**2 0 2 5**

( CBCS )

( 2nd Semester )

# MATHEMATICS

SECOND PAPER

**( Algebra )**

*Full Marks* : 75

*Time* : 3 hours

*The figures in the margin indicate full marks for the questions*

## ( SECTION : A—OBJECTIVE )

( *Marks* : 10 )

Tick (✓) the correct answer in the brackets provided : 1×10=10

**1.** The number of commutative binary composition on a finite set A having $n$ elements is

(a) $n^2$ ( )

(b) $n^{n^2}$ ( )

(c) $\frac{n^2 \quad n}{2}$ ( )

(d) $n$ ( )

**2.** defined by $a \quad b \quad \frac{ab}{2}$ is a binary operation on $Q$, the set of rational numbers. Here, the inverse element of $a$ is

(a) $\frac{4}{a}$ ( )

(b) 1 ( )

(c) $\frac{a}{4}$ ( )

(d) 0 ( )

**3.** The remainder on dividing $11^7$ by 18 is

(a) 12 ( )

(b) 11 ( )

(c) 9 ( )

(d) 1 ( )

**4.** The number of generators of a cyclic group of order 10 is

(a) 5 ( )

(b) 8 ( )

(c) 4 ( )

(d) 7 ( )

**5.** Every equation of an odd degree has at least

(a) 1 real root ( )

(b) 2 roots ( )

(c) 3 real roots ( )

(d) None of the above ( )

**6.** The expression $x^5 \quad 61x \quad p$ is divisible by $(x \quad 1)$, then the value of $p$ is

(a) 62 ( )

(b) 60 ( )

(c) –60 ( )

(d) 6 ( )

**7.** For the equation $x^3 \quad 7x^2 \quad 15x \quad 9 \quad 0$

(a) 2 is a root of multiplicity 2 ( )

(b) 2 is a root of multiplicity 1 ( )

(c) 3 is a root of multiplicity 3 ( )

(d) 3 is a root of multiplicity 2 ( )

**8.** The equation $x^5 \quad 6x \quad 2 \quad 0$ has

*(a)* three positive roots ( )

*(b)* one positive root which lies between 0 and 1 ( )

*(c)* no positive root ( )

*(d)* one negative root which lies between 1 and 2 ( )

**9.** If the sum of two roots of the equation $x^3 \quad 5x^2 \quad 16x \quad p \quad 0$ is zero, then the value of $p$ is

*(a)* 0 ( )

*(b)* 16 ( )

*(c)* 80 ( )

*(d)* None of the above ( )

**10.** $(1 \quad i)$ in De Moivre's form is

*(a)* $\sqrt{2}\ \cos\frac{\ }{3} \quad i\sin\frac{\ }{3}$ ( )

*(b)* $\sqrt{2}\ \cos\frac{5}{4} \quad i\sin\frac{5}{4}$ ( )

*(c)* $\sqrt{2}\ \cos\frac{\ }{4} \quad i\sin\frac{\ }{4}$ ( )

*(d)* None of the above ( )

## ( SECTION : B—SHORT ANSWERS )

( *Marks* : 15 )

Answer the following : 3×5=15

### UNIT—I

**1.** In the symmetric group of 3 symbols, prove that (12) (23) (132).

**OR**

**2.** Show that every cyclic group is an Abelian group.

### UNIT—II

**3.** Prove that every group of prime order is cyclic.

**OR**

**4.** Let $f : G \quad G$ be a group homomorphism. Then show that Ker $f \quad \{e\}$ iff $f$ is an isomorphism.

### UNIT—III

**5.** Expand $x^4 \quad 4x^3 \quad 3x^2 \quad 3x \quad 7$ in powers of $(x \quad 1)$.

**OR**

**6.** Show that $f(x) \quad x^2 \quad x \quad 1$ is irreducible over the field of rational numbers.

### UNIT—IV

**7.** Find the number of positive and negative real roots of the equation $x^5 \quad 6x \quad 2 \quad 0$. Also find the number of imaginary roots.

**OR**

**8.** If the equation $x^3 \quad 2x^2 \quad ax \quad b \quad 0$ has one of the roots as complex root $c + id$, then find the real root.

## UNIT—V

**9.** Find the condition that the equation $x^4 \quad px^3 \quad qx^2 \quad rx \quad s \quad 0$, may have two pairs of equal roots.

**OR**

**10.** Find all the values of $(1 \quad i)^{1/7}$ by De Moivre's theorem.

## ( SECTION : C—DESCRIPTIVE )

( *Marks* : 50 )

Answer the following : 10×5=50

## UNIT—I

**1.** *(a)* Prove that the set $G \quad \{0, 1, 2, 3, 4, 5\}$ is a finite Abelian group of order 6 w.r.t. addition modulo 6, using group table. 7

*(b)* If the element of a group $G$ is of order $n$, then show that $a^m \quad e$ iff $n$ is a divisor of $m$. 3

**OR**

**2.** *(a)* Find all distinct left cosets of the subgroup $H$ in the group $G$, where $H \quad \{e, (23)\}$, $G \quad S_3$ the symmetric group on 3 symbols. 6

*(b)* Show that the inverse of the product of two elements of a group $G$ is the product of the inverses taken in the reverse order. 4

## UNIT—II

**3.** *(a)* State and prove Lagrange's theorem. 1+4=5

*(b)* If $R$ is the additive group of real numbers and $R$ the multiplicative group of all positive real numbers, then show that the mapping $f : R \qquad R$ defined by $f(x) \quad \log x \quad x \quad R$ is an isomorphism. 5

**OR**

**4.** *(a)* State and prove Fermat's theorem. 1+5=6

*(b)* Show that a group of prime order has no proper subgroup. 4

UNIT—III

**5.** *(a)* If a polynomial $f(x)$ is divided by $(x \quad)(x \quad)$, , then prove that the remainder is $\frac{(x \quad)f(\ ) \quad (x \quad)f(\ )}{}$. 1+6=7

*(b)* Prove that $x^4 \quad x^2 \quad 1$ is a factor of $x^{12} \quad 1$. 3

**OR**

**6.** *(a)* If $p$ is prime, then show that the polynomial $(x) \quad x^{p \quad 1} \quad \cdots \quad x \quad 1$ is irreducible over the field of rational numbers $Q$. 6

*(b)* If a polynomial $f(x)$ of degree $n \quad 2$ is divisible by $(x \quad)^2$, then prove that the remainder is $(x \quad)f(\ ) \quad f(\ )$. 4

UNIT—IV

**7.** *(a)* State and prove fundamental theorem of algebra. 1+7=8

*(b)* Show that the equation $x^{12} \quad x^4 \quad x^3 \quad x^2 \quad 1 \quad 0$ has at least six complex roots. 2

**OR**

**8.** *(a)* Find the common roots of the equations $8x^3 \quad 24x^2 \quad 24x \quad 9 \quad 0$ and $8x^3 \quad 72x^2 \quad 96x \quad 45 \quad 0$ and hence solve the equations completely. 6

*(b)* If an equation $f(x) \quad 0$, whose coefficients are all real quantities, has a root of the form $\quad i$, then prove that the conjugate complex number $\quad i$ is also a root of the same equation. 4

## UNIT—V

**9.** *(a)* The equation $ax^3 \quad 3bx^2 \quad 3cx \quad d \quad 0$ has two equal roots. Prove that $(bc \quad ad)^2 \quad 4(b^2 \quad ac)(c^2 \quad bd)$. 6

*(b)* Solve

$$\left(\frac{1 \quad \cos \quad i\sin}{1 \quad \cos \quad i\sin}\right)^6$$

4

**OR**

**10.** *(a)* If $x \quad \frac{1}{x} \quad 2\cos$ , then show that $x^2 \quad \frac{1}{x^2} \quad 2\cos 2$ . 4

*(b)* Using Cardan's method, solve the equation $x^3 \quad 12x^2 \quad 6x \quad 10 \quad 0$. 6

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