MATH100 (MAJOR) **Student's Copy**

2 0 2 5

(NEP—2020)

(1st Semester)

MATHEMATICS (MAJOR)

(Vector Analysis)

Full Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

(SECTION : A—OBJECTIVE)

(*Marks* : 10)

Tick (✓) the correct answer in the brackets provided : 1×10=10

1. If $\vec{F} \quad t^2\hat{i} \quad \sin t\hat{j} \quad e^t\hat{k}$, then the value of $\frac{d\vec{F}}{dt}$ at $t \quad 0$ is

(a) $\hat{k}$ ( )

(b) $\hat{j} \quad \hat{k}$ ( )

(c) $2\hat{i} \quad \hat{j} \quad e^t\hat{k}$ ( )

(d) $\hat{i}$ ( )

2. The derivative of a constant vector field $\vec{A}$ is

(a) zero vector ( )

(b) magnitude of the vector field ( )

(c) a new vector field dependent on time ( )

(d) undefined ( )

3. For a vector field $\vec{A}(x, y, z)$, the partial derivative $\frac{\vec{A}}{x}$ applies to

(a) the x-component only ( )

(b) only components not dependent on x ( )

(c) the magnitude of the vector field ( )

(d) all components that depend on x ( )

4. The divergence of a vector field $\vec{F}$ is

(a) a scalar field ( )

(b) a vector field ( )

(c) a constant ( )

(d) None of the above ( )

5. The directional derivative of $(x, y) \quad x^2 \quad y^2$ at (2, 1) in the direction of $\hat{j}$ is

(a) 0 ( )

(b) 1 ( )

(c) 2 ( )

(d) 3 ( )

6. If is a scalar field, then the value of $\vec{\ } \; (\vec{\ })$ is

(a) ^{2} ( )

(b) 0 ( )

(c) $\vec{\ }(\vec{\ })\;\vec{\ }$ ( )

(d) $(\vec{\ })$ ( )

7. The surface integral $_S \vec{F} \; d\vec{S}$ computes

(a) the flux of $\vec{F}$ through the surface S ( )

(b) the circulation of $\vec{F}$ along the boundary of S ( )

(c) the divergence of $\vec{F}$ ( )

(d) the curl of $\vec{F}$ ( )

8. For a conservative vector field $\vec{F}$, the scalar potential function satisfies

(a) $\vec{F}$ $\rightarrow$ ( )

(b) $\vec{F}$ $\rightarrow$ ( )

(c) $\rightarrow$ $\vec{F}$ ( )

(d) $\rightarrow$ $\vec{F}$ 0 ( )

9. What does Gauss divergence theorem relate?

(a) Line integrals and surface integrals ( )

(b) Volume integrals and line integrals ( )

(c) Surface integrals and volume integrals ( )

(d) Surface integrals and curl ( )

10. Stokes' theorem is applicable when the surface is

(a) closed ( )

(b) infinite ( )

(c) defined in three dimensions ( )

(d) open with a boundary ( )

(SECTION : B—SHORT ANSWERS)

(*Marks* : 15)

Answer *five* questions, taking at least *one* from each Unit : 3×5=15

UNIT—I

1. If $\vec{F}(x, y) \quad (e^x)\hat{i} \quad (x^2 y \quad \cos y)\hat{j}$, then find the values of $\dfrac{\vec{F}}{x}$ and $\dfrac{\vec{F}}{y}$.

2. If $\vec{A} \quad (t \quad 1)\hat{i} \quad (t^2 \quad t \quad 1)\hat{j} \quad (t^3 \quad t^2 \quad 2)\hat{k}$, then find the value of $\dfrac{d^2\vec{A}}{dt^2}$.

UNIT—II

3. Show that $\vec{\ } \quad (\vec{r} \,/\, r^2) \quad r \quad {}^2$.

4. Suppose that $\vec{A}$ and $\vec{B}$ are irrotational. Prove that $\vec{A} \quad \vec{B}$ is solenoidal.

UNIT—III

5. Evaluate the volume integral $\quad_V x^2 dV$, where V is the cube defined by

$$0 \quad x \quad 2,\ 0 \quad y \quad 3,\ 0 \quad z \quad 4$$

6. Evaluate $\quad_C \vec{F} \quad d\vec{r}$, where C is the path along a straight line joining (0, 0) to (1, 1) and $\vec{F} \quad (3x^2 \quad 6y)\hat{i} \quad 14y\hat{j}$.

UNIT—IV

7. If S is any closed surface enclosing a volume V and $\vec{A} \quad ax\hat{i} \quad by\hat{j} \quad cz\hat{k}$, then prove that $\quad_S \vec{A} \quad \hat{n} dS \quad (a \quad b \quad c)V$.

8. Find the area of the circle $x^2 \quad y^2 \quad r^2$, using Green's theorem.

(SECTION : C—DESCRIPTIVE)

(*Marks* : 50)

Answer *five* questions, taking at least *one* from each Unit : 10×5=50

UNIT—I

1. *(a)* A particle moves along the curve $x \quad 2t^2, y \quad t^2 \quad 4t, z \quad 3t \quad 5$, where t is time. Find the scalar and vector components of its velocity and acceleration at time $t \quad 1$ in the direction $\hat{i} \quad 3\hat{j} \quad 2\hat{k}$. 5

(b) If $(x, y, z) \quad xy^2z$ and $\vec{A} \quad xz\hat{i} \quad xy^2\hat{j} \quad yz^2\hat{k}$, then find $\frac{3}{x^2 \quad z}(\quad \vec{A})$ at the point $(2, \quad 1, 1)$. 5

2. *(a)* If $\vec{A} \quad x\hat{i} \quad y\hat{j} \quad z\hat{k}$ and $\vec{B} \quad z\hat{i} \quad x\hat{j} \quad y\hat{k}$, then evaluate the differential of $(\vec{A} \quad \vec{B})$. 5

(b) Prove that a vector function $\vec{F}(t)$ is of constant length if and only if

$$\vec{F} \quad \frac{d\vec{F}}{dt} \quad 0$$

5

UNIT—II

3. *(a)* Find div$\vec{F}$ and curl$\vec{F}$, where $\vec{F} \quad \text{grad}\,(x^2 \quad y^2 \quad z^2 \quad 3xyz)$. 3

(b) Find the directional derivatives of $\quad x^2 \quad y^2 \quad 2z^2$ at the point $P(1, 2, 3)$ in the direction of the line PQ, where Q has coordinates $(5, 0, 4)$. 3

(c) Show that $\quad ^2(\log r) \quad \frac{1}{r^2}$. 4

4. *(a)* Find the direction along which the directional derivative of the function

$$x(y \quad z) \quad y(z \quad x) \quad z(x \quad y)$$

at the point (0, 1, 2). Also find the greatest directional derivative. 5

(b) If $\vec{A} \quad (y^2 \quad z^3, 2xy \quad 5z, 3xz^2 \quad 5y)$, then show that curl $\vec{A}$ 0 and find scalar function (x, y, z) such that $\vec{A}$ grad . 5

UNIT—III

5. *(a)* If $\vec{F} \quad (2x \quad y)\hat{i} \quad (3y \quad x)\hat{j}$, then evaluate $_C \vec{F} \; d\vec{r}$, where C is the curve in the xy-plane consisting of straight line from (0, 0) to (2, 0) and then to (3, 2). 5

(b) Find the total work done in moving a particle in a force field $\vec{F} \quad 3xy\hat{i} \quad 5z\hat{j} \quad 10x\hat{k}$ along the curve $x \quad t^2 \quad 1, y \quad 2t^2, z \quad t^3$ from t 1 to t 2. 5

6. *(a)* Evaluate $_S \vec{A} \quad \hat{n} dS$ where $\vec{A} \quad (x \quad y^2)\hat{i} \quad 2x\hat{j} \quad 2yz\hat{k}$ and S is the surface of the plane $2x \quad y \quad 2z \quad 6$ in the first octant. 5

(b) Calculate $_V \vec{F} \; dV$ where $\vec{F} \quad x^2\hat{i} \quad y\hat{j} \quad z\hat{k}$, and V is the region bounded by the surface $x \quad 0, y \quad 0, x \quad 1, y \quad x, z \quad 0, z \quad 1 \quad x$. 5

UNIT—IV

7. *(a)* Verify Green's theorem for the integral

$$\circ_C (xy \quad y^2)\, dx \quad x^2 dy$$

where C is the closed curve of the region R bounded by $y \quad x$; $y \quad x^2$. 5

(b) Using divergence theorem, evaluate $_S \vec{A} \;\; \hat{n} dS$ where

$$\vec{A} \quad x^3\hat{i} \quad y^3\hat{j} \quad z^3\hat{k}$$

and S is the surface of the sphere $x^2 \quad y^2 \quad z^2 \quad a^2$. 5

8. *(a)* Verify Stokes' theorem for

$$\vec{F} \quad (2x \quad y)\hat{i} \quad (yz^2)\hat{j} \quad (y^2 z)\hat{k}$$

where S is the upper half of the sphere $x^2 \quad y^2 \quad z^2 \quad 1$ and C is its boundary. 5

(b) Use divergence theorem to evaluate

$$_S(x\,dy\,dz \quad y\,dz\,dx \quad z\,dx\,dy)$$

where S is the portion of the plane $x \quad 2y \quad 3z \quad 6$ which lies in the first quadrant. 5

★ ★ ★