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(NEP-2020)

(1st Semester)

MATHEMATICS (MAJOR/MINOR)

(Calculus)

Full Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

(SECTION : A—OBJECTIVE)

(Marks : 10)

Tick (✓) the correct answer in the brackets provided :

1×10=10

1. The graph of the function $f(x) = |x^2 - 1|$ and $f(x) = x^2 - 1$ never touch in the interval

(a) $(-\infty, 0)$ ()

(b) $(-1, 1)$ ()

(c) $[-1, 1]$ ()

(d) $(0, \infty)$ ()

2. Choose the correct statement :

- I. A continuous real-valued function is bounded in a closed interval.
- II. A continuous real-valued function in $[a, b]$ attains its extreme values on $[a, b]$.

- (a) Only I is correct ()
- (b) Only II is correct ()
- (c) Both I and II are correct ()
- (d) Neither I nor II is correct ()

3. The function $f(x) = |x - 2|$ is

- (a) not bounded at $x = 2$ ()
- (b) not defined at $x = 2$ ()
- (c) not continuous at $x = 2$ ()
- (d) not differentiable at $x = 2$ ()

4. Choose the correct statement :

The Cauchy's mean value theorem is valid for the functions

I. $f(x) = 5x^{4/5}$

II. $f(x) = x^3$

in the interval $[-1, 1]$.

- (a) Only I is correct ()
- (b) Only II is correct ()
- (c) Both I and II are correct ()
- (d) Neither I nor II is correct ()

5. The function $f(x) = \log x$ can be expanded in the power of $(x - 1)$ by using

(a) Maclaurin's theorem ()

(b) Taylor's theorem ()

(c) Both (a) and (b) ()

(d) Neither (a) nor (b) ()

6. The integral $\int e^x [f(x) - f'(x)] dx$ equals

(a) $e^x f(x)$ ()

(b) $e^x f'(x)$ ()

(c) $e^{-x} f(x)$ ()

(d) $e^{-x} f'(x)$ ()

7. The formula for $\int \sec x dx$ is

(a) $\log \tan \frac{x}{4} + \frac{x}{4} + C$ ()

(b) $\log \tan \frac{x}{4} + \frac{x}{2} + C$ ()

(c) $\log \tan \frac{x}{4} + \frac{x}{4} + C$ ()

(d) $\log \tan \frac{x}{4} + \frac{x}{2} + C$ ()

8. Let $f(x, y) = \frac{x - y}{x^2 + y}$, when $x^2 + y = 0$. Then $\lim_{(x, y) \rightarrow (1, 1)} f(x, y)$ along the line $x - 1 = 0$ is equal to

(a) -1 ()

(b) 0 ()

(c) 1 ()

(d) Does not exist ()

9. Choose the correct statement :

I. Existence of double limit does not follow repeated limit.

II. Existence of repeated limit always follow double limit.

(a) Only I is correct ()

(b) Only II is correct ()

(c) Both I and II are correct ()

(d) Neither I nor II is correct ()

10. The sequence $\frac{1}{n!}$ is

(a) bounded below but not above ()

(b) bounded above but not below ()

(c) monotonic decreasing ()

(d) monotonic increasing ()

(SECTION : B—SHORT ANSWERS)

(Marks : 15)

Answer *five* questions, taking at least *one* from each Unit :

3×5=15

UNIT—I

1. Discuss the derivability of $f(x) = |x|$ at the origin.
2. Find $\frac{d^n f(x)}{dx^n}$ for $f(x) = x^2 e^{-x}$, where n is constant.

UNIT—II

3. Expand e^x in the power of x up to 5.
4. State Cauchy's mean value theorem.

UNIT—III

5. Find the reduction formula for $\int \tan^n x \, dx$.
6. Using Leibniz's rule find $\frac{df(x)}{dx}$ for $f(x) = \int_x^{x^2} e^t \, dt$.

UNIT—IV

7. Show that the sequence $\{f_n\}$ defined by $f_n = 1 - \frac{1}{2} + \frac{1}{3} - \dots + \frac{1}{n}$ cannot be converge.
8. Show that the series $1 - \frac{1}{2} + \frac{1}{3} - \dots$ is not convergent.

(SECTION : C—DESCRIPTIVE)

(Marks : 50)

Answer five questions, taking at least one from each Unit :

10×5=50

UNIT—I

1. (a) Evaluate : 4
- (i) $\lim_{x \rightarrow \infty} \frac{x^4}{e^x}$
- (ii) $\frac{dy}{dx}$ for $x = \cos t$ and $y = \sin t$
- (b) Draw the graph of the function $f(x) = |1 - x| + |1 + x|$. Discuss the continuity and derivability at $x = 1$ and $x = -1$. 6
2. (a) Using ϵ - δ definition, show that $\lim_{x \rightarrow 0} (1 + e^{1/x})^{-1} = 1$. 5
- (b) Show that a function which is derivable at a point is necessarily continuous but not conversely. 5

UNIT—II

3. (a) State and prove Rolle's theorem. 7
- (b) Verify the validity of Rolle's theorem for $f(x) = \tan x$ in the interval $[0, \pi]$. 3
4. (a) State and prove Taylor's theorem with remainder in Cauchy's form. 7
- (b) Expand e^x in an infinite series in power of x . 3

UNIT—III

5. (a) Evaluate : (i) $\int_0^1 x^2 dx$ as a limit of sum and (ii) $\int_1^1 |x| dx$. 4
- (b) Obtain reduction formula for $\int_0^{\pi/2} \sin^n x dx$. 6
6. If $I_n = \int_0^{\pi/4} \tan^n x dx$, prove that $n(I_{n-1} - I_{n+1}) = 1$. Using this relation, evaluate $\int_0^{\pi/4} \tan^8 x dx$. 10

UNIT—IV

7. (a) Let $u = f(x, y)$ be a homogeneous function of n -degree, then show that

$$x \frac{u}{x} + y \frac{u}{y} = nu \quad 5$$

- (b) Define harmonic function. Prove that $u(x, y) = e^x \cos y$ is harmonic in x and y . 5

8. (a) Prove that if a sequence is convergent, then it converges to unique limit. 5

- (b) Prove that a positive term series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if and only if $p > 1$, and the series diverges for $p \leq 1$. 5
