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(NEP—2020)

(2nd Semester)

MATHEMATICS (MAJOR)

(Elementary Number Theory)

Full Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

(SECTION : A—OBJECTIVE)

(*Marks* : 10)

Tick (✓) the correct answer in the brackets provided : 1×10=10

1. The greatest common divisor of 24531 and 435 is

(a) 3 (   )

(b) 4 (   )

(c) 5 (   )

(d) 7 (   )

2. The least common multiple of 3562 and 273 is

(a) 74724 (   )

(b) 3562 (   )

(c) 74725 (   )

(d) 76748 (   )

3. A solution of the linear congruence $7x$ 5 (mod 8) is

(a) x 2 (mod 8) ( )

(b) x 3 (mod 8) ( )

(c) x 4 (mod 8) ( )

(d) x 5 (mod 8) ( )

4. Which of the following is complete residue system modulo 11?

(a) 0, 2, 12, 3, 15, 19, 21, 9, 7, 16, 17 ( )

(b) 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22 ( )

(c) – 4, – 3, – 2, – 1, 0, 1, 2, 3, 4, 5, 6 ( )

(d) 0, 1, 2, 3, 4, 5, 14, 15, 16, 17, 18 ( )

5. The remainder when 8^{103} is divided by 103 is

(a) 0 ( )

(b) 1 ( )

(c) 103 ( )

(d) 8 ( )

6. Which of the following is true?

(a) For any integer n 2, Euler's function (n) is even ( )

(b) For any integer n 2, Euler's function (n) is odd ( )

(c) For any integer n 2, Euler's function (n) is rational number ( )

(d) For any integer n 2, Euler's function (n) is zero ( )

7. The congruence x^2 x 4 0 (mod 5) has

(a) no solution ( )

(b) one solution ( )

(c) two solutions ( )

(d) four solutions ( )

8. The value of Legendre symbol of $\frac{29}{53}$ is

(a) – 1 ( )

(b) 1 ( )

(c) 0 ( )

(d) 2 ( )

9. Which Fibonacci number is known as the golden ratio, denoted by the Greek letter phi( )?

(a) 1·317 ( )

(b) 1·318 ( )

(c) 0·617 ( )

(d) 1·618 ( )

10. If n 7056, then the number of positive divisors of n, (n) is

(a) 1 ( )

(b) 0 ( )

(c) 45 ( )

(d) 90 ( )

(SECTION : B—SHORT ANSWERS)

(*Marks* : 15)

Answer *five* questions, taking at least *one* from each Unit : 3×5=15

UNIT—I

1. Prove that if $a \mid b$ and $b \mid c$, then $a \mid c$.

2. Prove that the number of primes is infinite.

UNIT—II

3. If p is a positive prime and n is any positive integer, prove that

$$(1) \quad (p) \quad (p^2) \quad \ldots \quad (p^{n \quad 1}) \quad (p^n) \quad p^n$$

4. Define complete residue system and reduced residue system with example.

UNIT—III

5. Find all odd primes p such that 3 is a quadratic residue modulo p.

6. Let p be an odd prime. Prove that

$$\frac{ab}{p} \qquad \frac{a}{p} \quad \frac{b}{p}$$

UNIT—IV

7. Prove that the Mobius function is multiplicative.

8. Prove that any two consecutive terms of Fibonacci sequence are relatively prime.

(SECTION : C—DESCRIPTIVE)

(*Marks* : 50)

Answer *five* questions, taking at least *one* from each Unit : 10×5=50

UNIT—I

1. *(a)* State and prove division algorithm. 1+5=6

(b) Find the number of distinct positive integral divisors and their sum for the integer 56700. 4

2. *(a)* Let a and b integers, not both zero and let d be a positive integer. Prove that d gcd (a, b) iff d satisfies *(i)* $d \mid a$ and $d \mid b$, and *(ii)* $c \mid a$ and $c \mid b$, then $c \mid d$. 6

(b) For positive integers a and b, prove that gcd (a, b) lcm (a, b) ab. 4

UNIT—II

3. *(a)* Prove that the congruence ax b (mod m) has a solution if and only if the greatest common divisor of a and m divides b, i.e., (gcd $(a, m) \mid b$). 6

(b) Show that 16! 86 is divisible by 323 by using Wilson's theorem. 4

4. *(a)* State and prove Fermat's theorem. 1+5=6

(b) Solve the linear congruence

$$13x \quad 9 \pmod{25}$$ 4

UNIT—III

5. *(a)* Let p be an odd prime and gcd (a, p) 1. Prove that a is a quadratic residue of p if and only if

$$a^{\left(\frac{p \quad 1}{2}\right)} \quad 1 \pmod{p}$$ 6

(b) Solve $353x \quad 254 \pmod{400}$. 4

6. *(a)* Using Chinese remainder theorem, solve the following systems of equations : 6

$$x \quad 3 \pmod{11}$$
$$x \quad 5 \pmod{19}$$
$$x \quad 10 \pmod{29}$$

(b) Find all solutions of $x^2 \quad x \quad 7 \quad 0 \pmod{3}$. 4

UNIT—IV

7. *(a)* Let p be a prime and n a positive integer. Prove that the exponent e such that $p^e \mid n!$ is

$$_{k \quad 1} \frac{n}{p^k}$$ 6

(b) If f is a multiplicative arithmetic function and F is defined by $F(n) \quad _{d|n} f(d)$, prove that F is also multiplicative. 4

8. *(a)* Let F and f be two arithmetic functions related by the equality

$$f(n) \quad _{\frac{a}{n}} \quad (a) F \quad \frac{n}{a}$$

Prove that $F(n) \quad _{a|n} f(a)$. 6

(b) For each integer $n \quad 1$, prove that

$$_{d|n} \quad (d) \quad \begin{matrix} 1 & \text{if} & n & 1 \\ 0 & \text{if} & n & 1 \end{matrix}$$ 4

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