**2 0 2 5**

( NEP—2020 )

( 2nd Semester )

## MATHEMATICS

( Major/Minor )

**( Algebra )**

*Full Marks* : 75

*Time* : 3 hours

*The figures in the margin indicate full marks for the questions*

**( SECTION : A—OBJECTIVE )**

( *Marks* : 10 )

Tick (✓) the correct answer in the brackets provided : 1×10=10

**1.** If $f(x)$ is divided by $ax \quad b$, then the remainder is

(a) $f \frac{b}{a}$ ( )

(b) $f \quad \frac{b}{a}$ ( )

(c) $f \frac{a}{b}$ ( )

(d) 0 ( )

**2.** If the expression $x^5 \quad 61x \quad p$ is divisible by $(x \quad 1)$, then the value of $p$ is

*(a)* 62 ( ) *(b)* 60 ( )

*(c)* 60 ( ) *(d)* 6 ( )

**3.** If a polynomial $f(x)$ of degree $n \quad 2$ is divisible by $(x \quad )^2$, then the remainder is

*(a)* $(x \quad )f(\ ) \quad f(\ )$ ( )

*(b)* $(x \quad )f(\ ) \quad f(x)$ ( )

*(c)* $(x \quad )f(\ ) \quad f(\ )$ ( )

*(d)* $(x \quad )f(\ )$ ( )

**4.** For the equation $x^3 \quad 7x^2 \quad 15x \quad 9 \quad 0$

*(a)* 2 is a root of multiplicity 2 ( )

*(b)* 3 is a root of multiplicity 1 ( )

*(c)* 2 is a root of multiplicity 3 ( )

*(d)* 3 is a root of multiplicity 2 ( )

**5.** Every equation of an odd degree has at least

*(a)* 1 real root ( )

*(b)* 2 real roots ( )

*(c)* 3 real roots ( )

*(d)* None of the above ( )

**6.** The equation $4x^3 \quad 13x^2 \quad 31x \quad 41 \quad 0$ has

*(a)* three positive roots ( )

*(b)* one positive root which lies between 0 and 1 ( )

*(c)* no positive root ( )

*(d)* only one positive root which lies between 1 and 2 ( )

**7.** The equation $x^4 \quad 2x^3 \quad 1 \quad 0$ has

(a) at least two imaginary roots ( )

(b) more than one positive root ( )

(c) more than one negative root ( )

(d) four real roots ( )

**8.** The sum of two roots of the equation $x^3 \quad px^2 \quad qx \quad r \quad 0$ is zero, then

(a) $p \quad q$ ( )

(b) $pr \quad q$ ( )

(c) $pq \quad r$ ( )

(d) $pq \quad r \quad 0$ ( )

**9.** The cube roots of unity are

(a) $2, 2\ , 2\ ^2$ ( )

(b) $1, \ , \ ^2$ ( )

(c) $2, \quad 2\ , \quad 2\ ^2$ ( )

(d) None of the above ( )

**10.** The complex number (3 4*i*) in De Moivre's form is

(a) 5(cos *i* sin ) ( )

(b) 5(cos *i* sin ) ( )

(c) 3(cos *i* sin ) ( )

(d) 4(cos *i* sin ) ( )

## ( SECTION : B—SHORT ANSWERS )

( *Marks* : 15 )

Answer *five* questions, taking at least *one* from each Unit : 3×5=15

### UNIT—I

**1.** Expand $x^4 \quad 4x^3 \quad 3x \quad 7$ in powers of $(x \quad 1)$.

**2.** Use the remainder theorem to find the remainder if $f(x) \quad 4x^5 \quad 3x^3 \quad 6x^2 \quad 5$ is divided by $(2x \quad 1)$.

### UNIT—II

**3.** Show that 2 is a multiple root of $x^3 \quad x^2 \quad 16x \quad 20 \quad 0$ with multiplicity two.

**4.** If an equation $f(x) \quad 0$, whose coefficients are all real quantities, has a root of the form $\quad i$ , then prove that the conjugate complex number $\quad i$ is also a root of the same equation.

### UNIT—III

**5.** Find the range of values of $k$ for which the equation $x^4 \quad 4x^3 \quad 8x^2 \quad k \quad 0$ has all real roots.

**6.** If $\ , \ ,$ are the roots of $x^3 \quad px^2 \quad qx \quad r \quad 0$, then evaluate $( \quad )( \quad )( \quad )$.

### UNIT—IV

**7.** If $n$ is a positive integer, then prove that

$$(\sqrt{3} \quad 1)^n \quad (\sqrt{3} \quad 1)^n \quad 2^{n \quad 1}.\cos \frac{n}{6}$$

**8.** Express $1 \quad i$ in De Moivre's form and find the fifth root of the number.

## ( SECTION : C—DESCRIPTIVE )

( *Marks* : 50 )

Answer *five* questions, taking at least *one* from each Unit : 10×5=50

### UNIT—I

**1.** *(a)* If a polynomial $f(x)$ is divided by $(x \quad)(x \quad)$, , then prove that the remainder is

$$\frac{(x \quad)f(\quad) \quad (x \quad)f(\quad)}{}$$

6

*(b)* Using Eisenstein's irreducibility criterion, show that

$$f(x) \quad 3x^4 \quad 15x^2 \quad 10$$

is irreducible over the field of rational numbers. 4

**2.** *(a)* State and prove Division Algorithm. 1+6=7

*(b)* Show that $f(x) \quad x^2 \quad x \quad 2$ is irreducible over the field of rational numbers. 3

### UNIT—II

**3.** *(a)* State and prove fundamental theorem of algebra. 1+7=8

*(b)* Prove that irrational roots occur in conjugate pairs. 2

**4.** *(a)* Find the common roots of the equation $2x^3 \quad 7x^2 \quad 14x \quad 12 \quad 0$ and $4x^3 \quad 5x^2 \quad 10x \quad 12 \quad 0$ and hence solve the equations completely. 6

*(b)* If the equation $x^3 \quad 2x^2 \quad ax \quad b \quad 0$ has one of the roots as complex root $c \quad id$, then find the real root. 4

## UNIT—III

**5.** *(a)* Find the necessary condition for the roots of the equation $x^3 \quad px^2 \quad qx \quad r \quad 0$ to be in—

*(i)* arithmetic progression;

*(ii)* geometric progression;

*(iii)* harmonic progression. 6

*(b)* Diminish the roots of the equation $x^4 \quad 4x^3 \quad 3x^2 \quad 3x \quad 7 \quad 0$ by 1. 4

**6.** *(a)* If the polynomial $x^4 \quad px^2 \quad qx \quad r$ has a factor of the form $(x \quad )^3$, then show that $8p^3 \quad 27q^2 \quad 0$. 4

*(b)* If , , are the roots of $x^3 \quad px^2 \quad qx \quad r \quad 0$, then find the equation whose roots are $\frac{1}{\ }, \quad \frac{1}{\ }, \quad \frac{1}{\ }$. 6

## UNIT—IV

**7.** *(a)* Express

$$\frac{(\cos 3 \quad i \sin 3 \ )^5}{(\cos \quad i \sin \ )^6}$$

in the form $(a \quad ib)$. 3

*(b)* Using Cardan's method, solve the equation $x^3 \quad 6x \quad 2 \quad 0$. 7

**8.** *(a)* Solve $z^5 \quad 1 \quad 0$ by De Moivre's theorem. 4

*(b)* Deduce Cardan's method of solution of cubic equation. 6

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