**2 0 2 5**

( NEP—2020 )

( 4th Semester )

## MATHEMATICS (MAJOR/MINOR)

### ( Numerical Analysis—I )

*Full Marks* : 75

*Time* : 3 hours

*The figures in the margin indicate full marks for the questions*

### ( SECTION : A—OBJECTIVE )

( *Marks* : 10 )

Put a Tick ☑ mark against the correct answer in the boxes provided : 1×10=10

**1.** A factorial function denoted by $x^{(n)}$ where $n$ is a positive integer is given by

(a) $x(x\ \ h)(x\ \ 2h)(x\ \ 3h)\ldots[x\ \ (n\ \ 1)h]$ ☐

(b) $(x\ \ h)(x\ \ 2h)(x\ \ 3h)\ldots(x\ \ nh)$ ☐

(c) $(x\ \ h)(x\ \ 2h)(x\ \ 3h)\ldots(x\ \ (n\ \ 1)h)$ ☐

(d) $x(x\ \ h)(x\ \ 2h)(x\ \ 3h)\ldots(x\ \ nh)$ ☐

**2.** Which of the following expressions is incorrect?

(a) $y_{3/2} \quad y_2 \quad y_1$ ☐

(b) $f(x) \quad f \quad x \quad \frac{h}{2} \quad f \quad x \quad \frac{h}{2}$ ☐

(c) $f(x) \quad \dfrac{f \quad x \quad \frac{h}{2} \quad f \quad x \quad \frac{h}{2}}{2}$ ☐

(d) $E^n f(x) \quad f(x \quad nh)$ ☐

where $E$, , denote shift, central and average difference operators respectively.

**3.** Newton's formula for forward interpolation formula is given by

(a) $y \quad y_0 \quad u \; y_0 \quad \frac{u(u \quad 1)}{\underline{|2}} \quad {}^2y_0 \cdots$ where $u \quad (x \quad x_0) / h$ ☐

(b) $y \quad y_n \quad u \; y_n \quad \frac{u(u \quad 1)}{\underline{|2}} \quad {}^2y_n \cdots$ where $h \quad (x \quad x_n) / u$ ☐

(c) $y \quad y_0 \quad u \; y_0 \quad \frac{u(u \quad 1)}{\underline{|2}} \quad {}^2y_0 \cdots$ where $h \quad (x \quad x_0) / u$ ☐

(d) $y \quad y_0 \quad u \; y_0 \quad \frac{u(u \quad 1)}{\underline{|2}} \quad {}^2y_0 \cdots$ where $u \quad (x \quad x_n) / h$ ☐

**4.** The relation between divided and simple difference given by the relation

(a) $(x_n, \cdots, x_2, x_1, x_0) \quad \frac{{}^n y_1}{n! h^n}$ ☐

(b) $(x_n, \cdots, x_2, x_1, x_0) \quad \frac{{}^n y_1}{(n \quad 1)! h^n}$ ☐

(c) $(x_n, \cdots, x_2, x_1, x_0) \quad \frac{{}^n y_0}{(n \quad 1)! h^n}$ ☐

(d) $(x_n, \cdots, x_2, x_1, x_0) \quad \frac{{}^n y_0}{n! h^n}$ ☐

**5.** The method for obtaining the solution of a system of simultaneous equation by Gauss-Jordan elimination method depends on reducing the coefficient matrix to a/an

*(a)* diagonal matrix ☐

*(b)* upper triangular matrix ☐

*(c)* lower triangular matrix ☐

*(d)* diagonally dominant matrix ☐

**6.** Solving the system of simultaneous equations in $n$ unknowns by Crout's method, the method involves

1. diagonal matrix
2. upper and lower triangular matrix
3. forward and backward substitution
4. diagonally dominant matrix

Select the correct answer.

*(a)* 1, 2 and 3 ☐

*(b)* 1 and 2 only ☐

*(c)* 2 and 3 only ☐

*(d)* 1 and 4 ☐

**7.** If $f(x)$ $x^2$, then the divided difference of (2,4,9) is equal to

*(a)* 1 ☐

*(b)* 1 ☐

*(c)* 0 ☐

*(d)* 2 ☐

**8.** The value of , where , denote the forward and backward difference operator respectively, is

(a) ☐

(b) ☐

(c) ☐

(d) ☐

**9.** For solving ordinary differential equation numerically, the most reliable and most accurate among the following is

(a) Taylor's method ☐

(b) Picard's method ☐

(c) Euler's method ☐

(d) Runge-Kutta method ☐

**10.** Taylor series method is a powerful single step method if we are able to find easily the successive

(a) derivatives ☐

(b) integration ☐

(c) continuity ☐

(d) partial derivatives ☐

## ( SECTION : B—SHORT ANSWERS )

( *Marks* : 15 )

Answer *five* questions, taking at least *one* from each Unit : 3×5=15

UNIT—I

**1.** Can we find a real root of the equation $x^3 \quad x^2 \quad 1 \quad 0$ in the interval $[0, 1]$ by the method of successive iteration? Give a brief justification.

**2.** Express $\ ^3 y_1$ in terms of the lower order difference resulting to dependent variable (entry) only.

UNIT—II

**3.** For the function $f(a) \quad \frac{1}{a^2}$, find the divided difference of $\ (u, v, w)$.

**4.** Show that the divided differences are independent of the order of arguments, i.e., $\ (x_0, x_1, x_2) \quad\ (x_2, x_1, x_0)$.

UNIT—III

**5.** Solve the given equation by Gauss-Jordan method $x \quad 2y \quad 4,\ x \quad y \quad 13$.

**6.** Check whether the system of equations is a diagonally dominant system. If not make it a diagonally dominant system and write the corresponding system of equations :

$$3x \quad 9y \quad 2z \quad 10$$
$$4x \quad 2y \quad 13z \quad 19$$
$$4x \quad 2y \quad z \quad 3$$

UNIT—IV

**7.** Obtain the Picard's iteration formula for solving differential equation $\frac{dy}{dx} \quad f(x, y)$ numerically with the initial condition $y \quad y_0$ when $x \quad x_0$.

**8.** Using Taylor's method, find $y(0\ 1)$ by solving the differential equation $\frac{dy}{dx} \quad 1 \quad xy$ with $y(0) \quad 2$.

## ( SECTION : C—DESCRIPTIVE )

( *Marks* : 50 )

Answer *five* questions, taking at least *one* from each Unit : 10×5=50

### UNIT—I

**1.** *(a)* Find the second difference of the polynomial $f(x) \quad x^4 \quad 12x^3 \quad 42x^2 \quad 30x \quad 9$, taking $h \quad 2$. Write the answer in normal polynomial expression. 4+1=5

*(b)* If $y \quad \frac{1}{x(x \quad 3)(x \quad 6)}$, then prove that $^2 y \quad \frac{108}{x(x \quad 3)(x \quad 6)(x \quad 9)(x \quad 12)}$. 5

**2.** *(a)* Let $f(x)$ be a polynomial of degree $n$. Then prove that the $n$th difference of $f(x)$ is a constant and all higher order differences are zero, i.e.,

$$^r[f(x)] \quad \begin{matrix} \text{constant} & \text{if} & r \quad n \\ 0 & \text{if} & r \quad n \end{matrix}$$

5

*(b)* Find the smallest positive root of $x^2 \quad \log_e x \quad 12 \quad 0$ by Regula-Falsi method. 5

### UNIT—II

**3.** *(a)* From the data given below, find the number of students whose weight is between 60 and 70 : 5

| Weight | 0–40 | 40–60 | 60–80 | 80–100 | 100–120 |
|---|---|---|---|---|---|
| No. of students | 250 | 120 | 100 | 70 | 50 |

*(b)* Obtain the Newton's formula for backward interpolation of a function with equal intervals of the argument. 5

**4.** *(a)* Obtain Newton's divided difference interpolation formula for non-equal intervals of the argument. 5

*(b)* Find the equation of the cubic curve which passes through the points $(4, \quad 43)$, $(7, 83)$, $(9, 327)$, $(12, 1053)$. Hence find $f(10)$. 5

## UNIT—III

**5.** *(a)* Solve the following system of equations by Gaussian elimination method : 5

$$3x \quad y \quad 2z \quad 3$$
$$2x \quad 3y \quad z \quad\quad 3$$
$$x \quad 2y \quad z \quad 4$$

*(b)* Solve the following system by Gauss-Jordan method : 5

$$10x \quad y \quad z \quad 12$$
$$2x \quad 10y \quad z \quad 13$$
$$x \quad y \quad 5z \quad 7$$

**6.** *(a)* Solve the following system of equations by Crout's method : 5

$x \quad y \quad z \quad 1$, $3x \quad y \quad 3z \quad 5$ and $x \quad 2y \quad 5z \quad 10$

*(b)* Solve the following system of equations by Gauss-Seidel method : 5

$$10x \quad 2y \quad z \quad 9$$
$$x \quad 10y \quad z \quad\quad 22$$
$$2x \quad 3y \quad 10z \quad 22$$

## UNIT—IV

**7.** *(a)* Solve $\frac{dy}{dx} \quad 1 \quad y,\ y(0) \quad 0$ using Euler's method to find $y(0\ \ 1)$ and $y(0\ \ 2)$. Compare the result with results of the exact solution. 4+1=5

*(b)* Compute $y(0\ \ 1)$ by Runge-Kutta method of second order for the differential equation $\frac{dy}{dx} \quad 1 \quad xy,\ y(0) \quad 2$. 5

**8.** *(a)* Using Taylor's method, find $y(0\ \ 1)$ correct to 3 decimal places from $\frac{dy}{dx} \quad 2xy \quad 1,\ y_0 \quad 0$. 4

*(b)* Given that $\frac{dy}{dx} \quad \frac{y}{x} \quad \frac{1}{x^2}$ with $y(1) \quad 1$. Evaluate $y(1\ \ 3)$ by modified Euler's method. 6

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