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(NEP—2020)

(5th Semester)

MATHEMATICS (MAJOR2)

(Analytical Geometry)

Full Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

(SECTION : A—OBJECTIVE)

(*Marks* : 10)

Tick (✓) the correct answer in the brackets provided : 1×10=10

1. If the pair of lines $x^2 \quad 2pxy \quad y^2 \quad 0$ and $x^2 \quad 2qxy \quad y^2 \quad 0$ be such that each pair bisects the angle between the other pair, then

(a) $p \quad q \quad 1$ ( )

(b) $p \quad q \quad 1$ ( )

(c) $pq \quad 1$ ( )

(d) $pq \quad 1$ ( )

2. The common tangent of the circle $x^2 \quad y^2 \quad 4ax \quad 0$ and the parabola $y^2 \quad 4ax$ is

(a) $x \quad a$ ( )

(b) $x \quad 0$ ( )

(c) $x \quad 4a$ ( )

(d) $x \quad a \quad 0$ ( )

3. The gradients m_1 and m_2 of the conjugate diameter of the ellipse $\frac{x^2}{a^2} \quad \frac{y^2}{b^2} \quad 1$ are related by

(a) $m_1 \quad m_2 \quad \frac{a^2}{b^2}$ ( )

(b) $m_1 \quad m_2 \quad \frac{b^2}{a^2}$ ( )

(c) $m_1 \quad m_2 \quad \frac{a^2}{b^2}$ ( )

(d) $m_1 \quad m_2 \quad \frac{b^2}{a^2}$ ( )

4. The conic $\frac{3}{r} \quad 4 \quad 4\cos$ represents

(a) parabola ( )

(b) ellipse ( )

(c) hyperbola ( )

(d) rectangular hyperbola ( )

5. The distance of the point (4, 3, 5) from *XZ*-plane is

(a) 3 ( )

(b) 5 ( )

(c) 7 ( )

(d) $5\sqrt{2}$ ( )

6. For what value of k the lines $\frac{x\ 1}{3}\ \ \frac{y\ 2}{2k}\ \ \frac{z\ 3}{2}$ and $\frac{x\ 1}{3k}\ \ y\ \ 5\ \ \frac{z\ 6}{5}$ will be perpendicular?

(a) 8 ( )

(b) $\frac{2}{3}$ ( )

(c) $\frac{10}{7}$ ( )

(d) $\frac{1}{2}$ ( )

7. The distance of the point (3, 2, 1) from the line $\frac{x\ 1}{3}\ \ \frac{y}{4}\ \ \frac{z\ 3}{1}$ is

(a) $\sqrt{17}$ ( )

(b) $\sqrt{\frac{5}{2}}$ ( )

(c) 8 ( )

(d) 13 ( )

8. The equation of sphere passing through origin and having intercept parts a, b and c with coordinate axes is

(a) $x^2 \quad y^2 \quad z^2 \quad ax \quad by \quad cz \quad 0$ ( )

(b) $x^2 \quad y^2 \quad z^2 \quad 2ax \quad 2by \quad 2cz \quad 0$ ( )

(c) $x^2 \quad y^2 \quad z^2 \quad a \quad b \quad c$ ( )

(d) $x^2 \quad y^2 \quad z^2 \quad ax \quad by \quad cz \quad 0$ ( )

9. The axis of the cylinder $f(x, y) \quad 0$ whose guiding curve is $f(x, y) \quad 0, z \quad 0$ is

(a) X-axis ( )

(b) Y-axis ( )

(c) Z-axis ( )

(d) not defined ( )

10. The cone $ax^2 \quad by^2 \quad cz^2 \quad 2fyz \quad 2gzx \quad 2hxy \quad 0$ will have three mutually perpendicular tangent planes, if

(a) $a^2 \quad b^2 \quad c^2 \quad fg \quad gh \quad hf \quad 0$ ( )

(b) $f^2 \quad g^2 \quad h^2 \quad ab \quad bc \quad ac \quad 0$ ( )

(c) $a^2 \quad b^2 \quad c^2 \quad 3fgh$ ( )

(d) $f^2 \quad g^2 \quad h^2 \quad 3abc$ ( )

(SECTION : B—SHORT ANSWERS)

(*Marks* : 15)

Answer *five* questions, taking at least *one* from each Unit : 3×5=15

UNIT—I

1. Find the angle through which the axes are to be rotated so that the equation $3x^2 \quad 2xy \quad 3y^2 \quad 1 \quad 0$ may be reduced to the form $Ax^2 \quad By^2 \quad 1$. Also find A and B.

2. Find the equation of the pair of straight lines through the origin and perpendicular to the pair of straight lines given by $2x^2 \quad 5xy \quad 2y^2 \quad 10x \quad 5y \quad 0$.

UNIT—II

3. Find the locus of middle points of chords of the parabola $y^2 \quad 4ax$ which are passing through the focus.

4. Find the polar form of the left branch of the hyperbola $9x^2 \quad 16y^2 \quad 144$.

UNIT—III

5. Find the plane that bisects the acute angle between the planes $3x \quad 6y \quad 2z \quad 5 \quad 0$ and $4x \quad 12y \quad 3z \quad 3 \quad 0$.

6. Find the plane which contains the straight line $x \quad \frac{y \quad 3}{2} \quad \frac{z \quad 5}{3}$ and perpendicular to the plane $2x \quad 7y \quad 3z \quad 1$.

UNIT—IV

7. Obtain the equation of the circle lying on the sphere

$$x^2 \quad y^2 \quad z^2 \quad 2x \quad 2y \quad 4z \quad 3 \quad 0$$

and having its centre at the point (2, 2, –3).

8. Find the equation of the right circular cylinder of radius 2 and whose axis is the line $\frac{x \quad 1}{2} \quad \frac{y}{3} \quad \frac{z \quad 3}{1}$.

(SECTION : C—DESCRIPTIVE)

(*Marks* : 50)

Answer *five* questions, taking at least *one* from each Unit : 10×5=50

UNIT—I

1. *(a)* Prove that the general equation of second degree

$$ax^2 \quad 2hxy \quad by^2 \quad 2gx \quad 2fy \quad c \quad 0$$

will represent two parallel straight lines, if $h^2 \quad ab$ and $bg^2 \quad af^2$, and the distance between them is $2\sqrt{\frac{g^2 \quad ac}{a(a \quad b)}}$. 6

(b) Reduce the equation $7x^2 \quad 2xy \quad 7y^2 \quad 16x \quad 16y \quad 8 \quad 0$ to the standard form. 4

2. *(a)* Find the locus of the point of intersection of tangents of the ellipse which meet at right angle. 5

(b) If the normal to the rectangular hyperbola $xy \quad c^2$ at $ct, \frac{c}{t}$ meets the curve at $ct_1, \frac{c}{t_1}$, then show that $t^3t_1 \quad 1 \quad 0$. 5

UNIT—II

3. *(a)* If a pair of conjugate diameters cut the hyperbola and its conjugate at P, P and D, D respectively, then show that $CP^2 \quad CD^2 \quad a^2 \quad b^2$, where C is the centre of the hyperbola. 5

(b) If PSP and QSQ are two perpendicular focal chords of a conic, then prove that $\frac{1}{SP \quad SP} \quad \frac{1}{SQ \quad SQ}$ is constant. 5

4. *(a)* Find the equation of tangent to a conic in polar form. 5

(b) If the tangent at any point P of the conic $\frac{l}{r}$ 1 $e\cos$ meets the directrix at K, then show that the angle PSK is a right angle, where S is the focus of the conic. 5

UNIT—III

5. (a) If a plane meets the axes at A, B, C and the centroid of ABC is (, ,), then show that the equation of the plane is $\frac{x}{}$ $\frac{y}{}$ $\frac{z}{}$ 3. 5

(b) Find the equation of the plane which passes through (2, –3, 1) and is perpendicular to the line joining the points (3, 4, –1) and (2, –1, 5). 5

6. (a) Find the equation of the line through the point (2, 3, 1) and parallel to the planes $2x$ $3y$ $4z$ 5 and $3x$ $5y$ $4z$ 6. 5

(b) Find the magnitude and position of the shortest distance between the lines $\frac{x}{4}$ $\frac{y\ 1}{3}$ $\frac{z\ 2}{2}$ and $5x$ $2y$ $3z$ 6 0 x $3y$ $2z$ 3. 5

UNIT—IV

7. (a) Find the equation of a sphere which touches the sphere

$$x^2 \quad y^2 \quad z^2 \quad 2x \quad 6y \quad 1 \quad 0$$

at (1, 2, –2) and passes through the origin. 5

(b) Find the equation of the right circular cone with vertex at (2, 1, –3), semi-vertical angle is 45° and axis is parallel to OY. 5

8. (a) Find the angle between the lines of intersection of the plane x $3y$ z 0 and the cone x^2 $5y^2$ z^2 0. 5

(b) The plane $\frac{x}{a}$ $\frac{y}{b}$ $\frac{z}{c}$ 1 meets the coordinate axes at A, B, C. Prove that the equation of the cone generated by the line drawn from origin to meet the circle ABC is $yz\ \frac{b}{c}\ \frac{c}{b}$ $zx\ \frac{c}{a}\ \frac{a}{c}$ $xy\ \frac{a}{b}\ \frac{b}{a}$ 0. 5

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